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CHAPTER

6 Field and Fregean Platonism

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Abstract

This paper starts by offering a brief reconstruction of the Neo-Fregean approach as suggested in *Frege's Conception of Numbers as Objects* and distinguishes various challenges against the method of Abstraction. It then focuses on one line of criticism––Rejectionism––which is endorsed by Field in his Review of the previously mentioned book. The thought is to grant that the method of abstraction provides singular terms, however questions its ability to produce true statements. Furthermore, Field draws an analogy between the stipulation of Hume's Principle, which commits one to the existence of numbers and the ontological argument, which commits one to the existence of God. It is then shown that this analogy is amiss and that there is no real point of affinity with the Fregean platonist's ontological strategy and the ontological arguments. A further objection concerning the tacit ontological commitments on the right hand side of Abstraction Principle is discussed. The paper concludes considering 'the onus of proof' – issue for Nominalism-Platonism debates.

Keywords: abstract objects, abstraction, Hartry Field, Frege, Hume's Principle, Nominalism, ontological argument, Platonism, reference, Rejectionism

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In this paper, I want to return to an attempt I made a few years ago to bring general ideas about the nature of singular terms and singular reference to bear on the issue between mathematical platonism and its opponents. The argument in question took centre stage in my *Frege's Conception of Numbers as Objects* (henceforward *Frege's Conception*.)[1] It is at least implicit in Frege's way of proceeding in *Grundlagen*, and it is also to be found in various of Dummett's writings.[2] The argument has been deepened and strengthened in Hale's recent book.[3] It seems to me by far the best hope for a straightforward platonistic construal of large portions of classical mathematics, and my principal purpose here is to add to the reasons for confidence in it by responding to some criticisms of it recently canvassed by Field, in his review of *Frege's Conception*.[4]

1. Basics of Fregean Platonism

Frege's belief that numbers are objects is not to be dismissed as a technicality. It is the belief that numbers are objects in what is (or ought to be) the ordinary understanding of the term, and it is the product of a deceptively simple train of thought. Objects are what singular terms, in their most basic use, are apt to stand for. And they succeed in doing so when, so used, they feature in true statements. Certain sorts of expression, for instance the standard decimal numerals, and expressions formed by applying the numerical operator, 'the number of . . . ', to a predicate, are used as singular terms in the pure and applied arithmetical statements of identity and predication in which they feature. Many such statements are true. So such terms do have reference, and their reference is to objects.

The basic idea is that a reference is, as it were, imposed on a singular term by its occurrence in true contexts of an appropriate kind. It will be agreed, I imagine, that identity statements in which the term in question is one
p. 154 of the ↳ related terms, and predications in which it is the subject term, are of the 'appropriate kind' to subserve this thought. So the argument must succeed unless *either* the apparent singular terms of arithmetic do not really function as such *or* the apparently true 'appropriate' contexts in which they feature are not really true.

Clearly, the notion of a singular term appealed to by the argument must not in the first instance be explained by appeal to the idea of reference to objects. The *semantic* function of a singular term is—if successfully discharged—so to refer. But the argument requires this to be a consequence of a classification formulated differently. In *Frege's Conception* I made a case, following on earlier discussions by Dummett and Hale, for thinking that singular terms can be characterized by syntactic criteria.[5] Very roughly singular terms are marked off from others by the inferential liaisons of the statements in which they occur. The issue is, in fact, a fairly intricate one, but I shall be giving it no further attention here. Rather, I shall assume that such an account is available and that, by its lights, a large class of numerical expressions, no less than proper names standing for persons, towns, and rivers, and many ordinary definite descriptions and demonstratives, qualify as singular terms.

The syntactic criteria for singular termhood come into play, of course, only for expressions whose use is already established. We shall want to ask, for instance, whether numerical expressions do indeed feature in genuine identity statements (identified as such by proof-theoretic criteria), and whether they are so used as to sustain (first-order) existential generalization. If a given class of expressions pass such tests, the question must then arise how the use of the contexts in question is established. The 'deceptively simple' route to platonism requires the use of such contexts to have been established in such a way that we can indeed reasonably claim to recognize certain of them to be true. The essence of the arithmetical *logicism* proposed by Frege was that the use of arithmetical singular terms can be established by a programme first of contextual and then of explicit definitions of arithmetical vocabulary by means of logical vocabulary, a programme which, if successfully executed, would establish beyond doubt the epistemological pedigree of the basic laws of arithmetic.

As is well known, Frege's version of this programme turned out to be based on an incoherent notion of *extension of a concept* (or *course-of-values*.) One of the principal claims of *Frege's Conception* was that that is not the end of the matter. Logic may still provide the basis for an explanation of the concept of natural number from which the basic laws—the Peano axioms—follow. But the arithmetical case is complicated by the fact that, unless we follow Frege and presume ourselves to have certain 'logical objects' at our disposal, there is no hope of so defining arithmetical vocabulary that we can exhaustively eliminate it in use. Something less
p. 155 rigorous than eliminative ↳ definition is therefore all that can be demanded of the explanations which a workable version of arithmetical logicism has to provide. Rather than risk distraction by a group of rather subtle questions which now loom,[6] let us follow Frege's example[7] and concentrate, for the moment, on a simple case where the use of the controversial contexts can indeed be fully established by means of a programme of contextual definition.

The example is that of (a rather restrictive use of the notion of) *direction*. Suppose we have a first-order language containing, *inter alia*, a range of names, 'a', 'b', 'c', . . . , standing for members of a domain of straight lines (which are to be conceived, for these purposes, as concrete inscriptions), including the relation, ' . . . is parallel to . . . ' We proceed to introduce a singular term-forming operator on names of lines, 'D()', and a series of contextual definitions as follows:

(i) Any sentence of the form '$D(a_1) = D(a_2)$' is true if and only if 'a_1' and 'a_2' are names of lines and the lines they denote are parallel.

A range of other kinds of open sentence, 'φ[]', completable by direction-terms, are then introduced by reference to established predicates and relations on lines in accordance with the schema

(ii) 'φ D(x)' is true if and only if 'Fx' is true, where ' . . . is parallel to . . . ' is a congruence for 'F[]'.

Finally, we stipulate that

(iii) '(∃x)φ x' is true if and only if '(∃x)Fx' is true, where 'φ', and 'F' are as stipulated under (ii).

The effect of these stipulations is that we establish a simple 'language-game' of directions in which direction-terms satisfy any reasonable syntactic criteria for singular termhood. The claim of the sort of platonism with which we are concerned is then twofold. First, there is no sense in which, despite their satisfaction of these criteria, direction-terms might fail to be genuine—semantic—singular terms; second, that the contextual equivalences of statements in which they feature with statements of a (purportedly) unproblematic kind about lines embody a satisfactory account of how statements involving such terms may be known to be true and, hence, how knowledge of the existence of directions, and of their properties, is possible.

The argument is open, accordingly, to two different kinds of challenge. One challenge will dispute, in effect, that syntactic singular termhood suffices for semantic singular termhood: it will contend, that is, that an
p. 156 expression ↳ can pass as a singular term, by the syntactic criteria, without importing a specific ontological commitment into the truth-conditions of sentences in which it occurs. A familiar train of thought to this effect is that of the *ontological reductionism* criticized in *Frege's Conception*. The ontological reductionist contends that this is the fate of the direction-terms introduced by the above equivalences, and is shown to be so by the very equivalence of sentences in which they feature to sentences in which no reference to directions is made. The obvious reply[8] is that this way of looking at the equivalences presupposes that it is proper to take only their right-hand sides at face value, which is just what the platonist disputes. But a reductionist reading of such equivalences could be enforced if it turned out that reference to directions, qua abstract objects, is impossible since reference is essentially a *causally* constrained relation. An objection in the same spirit would be that reference, properly construed, requires *identifying knowledge* of the referent, and that such knowledge, too, is causally constrained. A third line of objection, advanced by Dummett,[9] is that it is proper to regard a (syntactic) singular term as genuinely referential only if the notion of reference plays an essential part in establishing its use—a part which it cannot play if that use is established by contextual definition.[10]

The reductionist tendency will, of course, typically manifest itself not in response to a proposed *introduction* of a class of singular terms, but in a programme for rendering untroublesome a range of already established contexts in which such (purportedly troublesome) singular terms occur. Still, one way or another, reductionism has to earn the right to read such equivalences in the manner it prefers. The platonist strategy, by contrast, will be to argue that there is no good cause to endow the equivalences with such a reductive significance—that, in particular, the objection of Dummett noted, and the objections issuing from causalist accounts of knowledge and reference, are misconceived—and that no good distinction can be drawn between an expression's functioning as a singular term according to syntactic criteria and its being appropriate to construe its semantics referentially.

Platonism and reductionism are united in their acceptance of the equivalences. They are also united in the belief that it is by reference to the epistemology of the right-hand sides, as ordinarily understood, that an account should proceed of how it is possible to know the left-hand sides. But the platonist sees the situation as constituting an explanation of how thought of and reference to abstract objects is unproblematic—at least for one who has no reservations about the content and knowability of the right-hand sides; whereas the reductionist regards the situation as demonstrating, rather, how p. 157 ↳ the comprehending use of the left-hand sides need involve neither thought of nor reference to abstract objects.

Set against both these views is (what I shall call) the *rejectionist* response. The rejectionist rejects the equivalences. He denies, that is, that any such equivalences can be uncovered by correct analysis of contexts already established in the language; and he denies that it is legitimate even to stipulate that such equivalences obtain by way of attempted explanation of the use of statements which purportedly involve reference to abstract objects. The second denial, in particular, may seem hard to comprehend: how can it be illicit to stipulate that the use of one class of contexts is to coincide with that of another, well-understood class? The answer is that the statements on the left-hand sides are not being regarded as utterly unstructured: rejectionism agrees with platonism that expressions of the form '$D(a_n)$', for instance, will meet the syntactic criteria for singular termhood if stipulation of the equivalences is allowed; and it is further agreed that no distinction is to be drawn between functioning by syntactic criteria as a singular term and importing commitment to an object. Rejectionism is thus the only outlet for an antiplatonist who judges that platonism wins its dispute with reductionism.

Field is a rejectionist.[11] He is content to accept that the notion of a singular term can be characterized syntactically, along the lines attempted in *Frege's Conception*, and that in the light of that characterization numerical expressions do indeed function as singular terms in arithmetical statements, whose truth demands, accordingly, the existence of numbers as objects. His question is: what reason do we have to think that any such statements are true?

2. Field's Rejectionism

Field makes heavy weather of discerning the response of *Frege's Conception* to this question. He quotes[12] the following passage from my book:

> Frege requires that there is no possibility that we might discard the preconceptions inbuilt into the syntax of our arithmetical language, and, the scales having dropped from our eyes, as it were, find that in reality there are no natural numbers, that in our old way of speaking we had not succeeded in referring to anything. Rather, it has to be the case that when it has been established, by the sort of syntactic criteria sketched, that a given class of terms are functioning as singular terms, and when it has been verified that certain appropriate sentences containing them are, by ordinary criteria, true, then it follows that those terms do genuinely refer. And being singular terms, their reference will be to objects. There is to be no further, intelligible question whether such terms really have a reference, whether there really *are* such objects.[13]

Field responds

> The kicker [*sic*] here is the phrase 'by ordinary criteria' . . . and proceeds to suggest, in effect, that the argument reduces to the claims that to function as a singular term by p. 158 ↳ the syntactic criteria sketched in *Frege's Conception* is to function as a genuine (semantic) singular term—which he accepts —and
>
> (S) what is true according to ordinary criteria really is true, and any doubts that this is so are

vacuous.[14]

Field proceeds to raise the obvious kind of query about (S): did the 'ordinary criteria' for truth in ancient Greece make 'Zeus is throwing thunderbolts' true whenever there was lightning? But the impression that such a question is pertinent is owing to a misreading of the quoted passage. The misreading is understandable, since the wording 'and when it has been verified that certain appropriate sentences containing them are, by ordinary criteria, true,' is ambiguous between a factive reading, when it is implied that the sentences in question have been verified *tout court*, and the non-factive reading, assumed by Field, under which all that has been verified is that the sentences meet ordinary criteria of truth, so that there is still apparent space for a question concerning the adequacy of those criteria. Recognizing that I intended to exclude that question, but assuming the non-factive reading, Field naturally found in the quoted passage an endorsement of the principle (S). But I intended the passage to be read the former way; verification that a class of sentences are, by ordinary criteria, true may not always be verification *tout court*; but it is so in the cases which interest us, in which there is, according to Fregean platonism, no issue concerning the adequacy of the criteria in question.

Say that a type of ground for a statement is *canonical* just in case the supposition that that statement is true entails the availability, at least in principle, of grounds of that type for believing it. Thus determining that all cats in Michigan weigh less than twenty pounds is supplying oneself with canonical grounds for the statement that all cats in the Great Lakes area weigh less than twenty pounds. So canonical grounds need not be conclusive. But they can be. If it is true that 241 × 73 = 17,593, a correctly conducted calculation will disclose as much. So correctly performing that calculation and getting the result, 17,593, is acquiring canonical grounds for the statement that 241 × 73 = 17,593. In these terms, the relevant points are two. First, lightning was not, presumably, even for the Greeks, a canonical ground for assertions about Zeus′ ballistic extravagances. Second, one line′s being parallel to another, for example, *is* a canonical ground for the identity of their directions and is, indeed, in the platonist view, a *conclusive* such ground.

This is the crux of the dispute. Field is prepared to allow that the direction-equivalences have a concept-fixing role. They constitute a theory in which a concept of direction is embodied, as contemporary physics constitutes a theory in which the concept of an electron is embodied. More than that, he is prepared to allow that the conditionalization of those equivalences on the supposition that directions do indeed exist *does* result in
p. 159 conceptual truths; it ↳ is a conceptual truth, for example, that if directions exist, then the directions of two lines are identical just in case those lines are parallel. But he insists that the concept so explained may—and actually does—apply to nothing. The theory can be—and is—false. He rejects the claim that the parallelism of two lines suffices a priori for the identity of their directions; what we can know a priori is only that it so suffices if they *have* directions. And he will take a parallel stance on the corresponding equivalences on which, for instance, a logicism of the sort adumbrated in *Frege's Conception* will base its account of number theory, and on all similar attempts to construe talk of abstract objects, mathematical or otherwise. The relevant kind of equivalences do indeed have an explanatory status; but it is an explanatory status which allows the possibility of their falsity.

In *Frege's Conception* I formulated a dilemma for anyone who would doubt the existence of any species of abstract object whose covering sortal concept is explained along the lines of the Fregean paradigm illustrated by direction.[15] The dilemma was simple: if, in accordance with the explanation, it is accepted that the obtaining of the relevant equivalence relation among items of the relevant, previously familiar kind suffices for identity under the new concept, then the reflexivity of that relation guarantees that the new sortal concept is instantiated. If 'a is parallel to b' is accepted as sufficient, in accordance with the explanation of direction, for the truth of 'The direction of a is identical with the direction of b', then the parallelism of every line to itself guarantees the self-identity of the associated direction and hence its existence. If, on the other hand, the obtaining of the relevant equivalence relation is not accepted as sufficient for identity under the new concept, then—since that was an integral part of the explanation of that concept 0—no doubt about the existence of that sort of thing can be intelligibly entertained; for there is no concept in terms of which to formulate the doubt.

Field's position seems almost to have been designed so as to allow him to dodge the horns of this dilemma: to insist both that parallelism does not suffice for identity of direction and that he *does* have a concept of direction —given by the equivalences—in terms of which to formulate the doubt about their existence. But there is at least prima-facie reason to suppose that the dilemma is still good. Reflect that in rejecting the platonist claim that the equivalences are true purely in virtue of their explanatory status, Field has thrown out the only extant operationally sufficient conditions for the truth of the claim that directions exist. But he has accepted that we understand the 'theory' of directions, and know, therefore, what it would be for directions to exist—and indeed that it is a conceptual truth that if directions exist, their identities and other characteristics may be determined in accordance with the equivalences. So we allegedly understand what it would be for directions to exist
p. 160 although we have no conception of a ground on which we could, in ↳ practice or in principle, rely in order to determine whether or not they do. What, then, does our understanding of the possibility consist in?

The trouble is that the explanatory challenges involved in this line of thought are ones which, unless they receive substantial further development, Field appears at liberty simply to decline. If he is charged that he owes a new account of what would suffice for the existence of directions, he can reply that he is under no obligation to provide such an account: that what would suffice for the existence of directions is only and precisely that. If we are inclined to press that, failing some such alternative account, it is quite unclear what it can be to understand the hypothesis that directions exist—a hypothesis which features in the conditionalized equivalences which Field accepts—Field can stonewall again, replying that that is simply the hypothesis that direction theory is true—a hypothesis which anyone will understand well enough who derives as much explanatory content from the direction equivalences as is legitimate. One feels the position is deeply unsatisfactory, but how is it to be assailed?

That is a difficult question to which I have attempted to respond elsewhere.[16] Here I wish to concentrate on the somewhat easier question whether Field succeeds in disclosing any actual *incoherence* in the platonist reading of the relevant kind of equivalences, or any other reason for regarding that reading as impermissible. Even if Field's position has the resources to defeat its critics, and his programme of nominalistic interpretation of physical theory can indeed be carried through, he cannot avoid the attribution of massive error to great sweeps of contemporary mathematical theory. Fregean platonism carries no such implication. So unless Field can show that it is actually illicit to treat the relevant kind of equivalences as, in the platonist manner, concept-explanatory *truths*, his programme must inevitably seem less attractive than the platonist programme.

3. Field's Objections

It is in Section 4 of his critical study of *Frege's Conception* that Field presents his objections to the Fregean species of platonism in general and the platonist treatment of the equivalences in particular. These seem to be two. The first is that it is not clear how, in contrast with its epistemologically more exotic relatives, Fregean platonism really can avoid the postulation of special intuitive or quasi-perceptual faculties, sensitive to abstract objects and their properties and relations. Second, Field is deeply suspicious of what he describes as

> The idea that the existence of objects can flow from an explanation of concepts . . . The idea that the existence of numbers flows from the explanation of the concept of number is reminiscent of the ontological argument for the existence of God, according to which it follows from the very concept of God that God exists.[17]

p. 161 His presentation of the first of these objections, and indeed much of the earlier part of his review, is gratuitously complicated by the presence of a character he calls the 'ontological inflationist'. The ontological inflationist is a kind of obverse of the reductionist who holds that the direction-terms occurring on the left-hand sides of the direction equivalences are not genuine singular terms. The inflationist, for his part, holds that the *line-terms* occurring on the right-hand sides are not genuine singular terms. Field seems to introduce this position only in order to make a meal of distinguishing it from Fregean platonism, although he realizes perfectly well that it is not available to someone who endorses the syntactic account of singular termhood on which Fregean platonism depends. Nevertheless the 'inflationist' and the platonist do have this in common: they agree that the existence of directions follows from the truth of statements on the right-hand sides. But if —and this is the objection—the claim that line a is parallel to line b, presupposes the existence of directions, why does the problematic epistemology of the latter not come across, as it were, to infect ordinary statements about lines and their properties and relations as well? No doubt the epistemology of the right-hand sides is in fact relatively unproblematic; but it is so—the objection continues—only because such statements are standardly taken *not* to import any claims about abstract objects; as soon as they are conceived as the platonist wishes, it is no longer clear that the routine verification-procedures associated with them suffice, and the appearance of unproblematic epistemology disappears.

The platonist claim is that statements on the right-hand sides entail that directions exist. The objection is that if they do, they inherit the epistemological difficulties associated with the existence of abstract objects. And the immediate reply should be that this is a *non-sequitur*: it cannot *always* be true that the consequences of a statement must be verified independently before that statement may be regarded as known—if it were, advancement of knowledge by inference would be impossible. Sometimes it is true; we might not be able to verify that Vicky is a vixen without first verifying that Vicky is female, for instance. But no reason has been produced for regarding the present kind of case as coming into that category, and it is reasonable to expect that none will be which does not beg the question against the platonist. For the platonist view, to stress, is that the concept of direction is *established* by reference to these equivalences; it is therefore out of the question that previously accepted verification-procedures for the right-hand sides could suddenly cease to be adequate as a result of their being saddled with a new kind of implication. Our very understanding of these implications, the platonist will insist, is dependent upon our fixing the concept of direction in such a way that statements about directions are verified by the *very same procedures* which we antecedently took to verify the corresponding statements about lines. To suppose that the ordinary procedures suffice for verification of the right-hand statements if but only if they are not construed as entailing the existence of directions is to assume—not argue p. 162 —that there is no sortal ↳ concept which can be constituted by the stipulation that the equivalences, under the ordinary understanding of their right-hand sides, are to hold.

Field proceeds to anticipate, via some skirmishes concerning whether or not the left-hand statements may be regarded as following *logically* from the right-hand statements, more or less this reply, and moves to his second objection, the mystery of the existence of objects 'flowing' from the explanation of concepts. He does not say, but does nothing to discourage his reader from thinking, that the Ontological Argument for the existence of God may stand or fall with platonism of the kind under consideration. 'Does Wright have anything to say', he in effect enquires, 'which makes Platonism look any more respectable than this suggestion would imply?'[18]

Well, yes I do. It needs emphasis, to begin with, that—with a qualification I shall make in the next paragraph—it is no part of Fregean platonism to regard the existence of the relevant species of abstract objects as entailed just by the way in which their covering sortal is explained. It is not, for instance, the way the concept of direction is explained—via stipulation of the equivalences—which entails that there are such things as directions, but those stipulations *together with* the truth of appropriate statements apt to feature on the right-hand sides. What does follow from the explanation is that any one of a relevant class of straight lines has a direction; but the existence of directions is contingent on the existence of members of that class. So the gap between such cases and the uncomfortable precedent of the Ontological Argument seems adequately broad.

It narrows, of course, when we move back to the case which the analogy with directions was mainly supposed to illuminate, namely the natural numbers. Now concepts replace straight lines and one–one correspondence replaces parallelism. And some of the right-hand statements in the relevant equivalences are theorems of second-order predicate logic with identity. Any concept, including all which may be defined purely logically, is put into one–one correspondence with itself by the relation of identity, for instance. So it is straightforward to prove, via

$$(N^{=}) \quad Nx{:}Fx = Nx{:}Gx \leftrightarrow (\exists R) \quad R \quad \text{1-1 correlates F with G},$$

that there is, for instance, a number of things which are self-identical, and a number of things which are not. Here the substance of what Field says is, for anyone disposed to accept the version of logicism for which I was arguing in *Frege's Conception*, correct. The existence of numbers follows from the nature of the concept of number—more accurately, from a statement of a canonical explanation of that concept—together with certain truths of logic, just as the existence of directions follows from a canonical explanation of the concept of
p. 163 direction together with certain truths about straight lines. But ↳ what follows from certain statements together with truths of logic follows from those statements *simpliciter*. So what Field says is true (or at least it may be true, if some of the difficulties, concerning impredicativity and other matters, which were canvassed in *Frege's Conception* can be satisfactorily met and the logicist's explanations are in good order). The existence of numbers, and indeed their satisfaction of the Peano axioms, flows out of the concept of number. But everything here is above board; why is this not a *congenial* discovery, rather than a cause for complaint about mystery?

Well, what about the comparison with the Ontological Argument? An immediate point of disanalogy is that that argument makes no attempt to fix the concept of God by associating the truth-conditions of statements concerning Him with those about any subject matter of an overtly non-theological sort about which we antecedently believe that a priori knowledge is possible. But the main point is that the Ontological Argument is flawed not in its very enterprise—the project of trying to establish an existential conclusion on a basis of conceptual reflection—but in detail. Briefly, since I have no wish to digress on the matter, it founders, *inter alia*, on the following dilemma. If the putative explanation of the concept of God, 'God is that than which nothing greater can be conceived', is represented as a universally quantified statement:

(i) For all x, x is identical with God if and only if no greater being than x can be conceived,

no contradiction follows from adding the supposition that there is no x identical with God. If, on the other hand, the explanation is couched like this

(ii) God = (ι x) no greater being than x can be conceived,

then it is consistent with assigning this statement a concept-fixing role to allow that it may be false—when no individual satisfies the description on the right-hand side. So, once again, the explanation fails to imply existence.

The latter, of course, is exactly what Field wants to say about the equivalences which, in the logicist programme, constitute the core of the explanation of the concept of number. But I am not, to repeat, concerned to question here whether it is coherent to take Field's view of them. The issue is whether Field has produced any independent argument why it is *impermissible* to view them in the platonist fashion to which he takes exception. And none seems to be in the offing. If the suggestion is, for instance, that once we reserve the right to stipulate that the numerical equivalences are true, we are powerless to refuse any proponent of the Ontological Argument the right to stipulate that (ii) is true, then the answer is that, whereas the first stipulation does no more than assign truth-conditions to statements of numerical identity, some of which it is then possible to show to be realized by accredited methods, the second would be an attempt to stipulate not truth-conditions but truth itself.

p. 164 I do not, by these remarks, mean to suggest either that nothing else is amiss with (many versions of) the Ontological Argument than its inability to avoid the dilemma mentioned, or that better versions of the argument are not available which do avoid that particular dilemma. What is clear is that the strategy of the argument has no real point of affinity with the Fregean platonist's ontological strategy. An ontological argument which did have such an affinity would proceed by contextual definitions of the truth-conditions of statements concerning God of such a kind that they could be established by the accredited methods associated with the *definientia*. No such argument is possible, one would imagine, for the straightforward reason that the cosmological implications of the *definienda* preclude their having any correct *definientia* with routinely accredited methods of verification.

4. Another Objection

There is, however, a third line of objection to the equivalences which Field does not explicitly offer. Indeed, it seems to me prima facie more powerful than those he does offer. The reductionist holds that the appearance of singular reference to directions on the left-hand sides of the relevant equivalences is misleading, that the ontological commitments of the left-hand-side statements are only and exactly what are suggested by the surface grammar of the right-hand sides. The Fregean platonist holds just the reverse: that the ontological commitments of the right-hand sides are just what are displayed in the surface grammar of the left-hand sides —an ontology of both directions and lines. But that seems to imply that even if we speak exclusively in the vocabulary of the right-hand sides, we nevertheless refer, willy-nilly, to directions as well. Indeed, this would be so even if we had no inkling of the concept of direction and never introduced direction terminology. For, as I wrote in *Frege's Conception*,

> [T]he absence of any *need* to construe natural number as a sortal concept would be completely irrelevant for 'ontotaxonomic' purposes. Availability is enough . . . The question, what kind of things are there, should not be approached by reference only to the sortal concepts which we need to employ for whatever purposes we happen to have, but by reference to all such concepts which admit of satisfactory explanation . . . if a sortal concept of natural numbers is available, and normal criteria determine that it has instances—that is, contexts of relevant types are true which contain terms purporting to denote natural numbers—then there *are* such things.[19]

The point is: it *is* a species of platonism with which we are here concerned. We do not create directions, or numbers, or sets by creating sortal concepts of direction, number, and set. When a sortal concept is coherently explicable, and statements purporting to involve reference to instances of it are verifiable in the light of
p. 165 canonical grounds associated with the concept, then it does ↳ indeed have instances; and it has them whether we choose to acknowledge the fact, or even arrive at any understanding of the concept in the first place. But is there not something absurd about the resulting situation? For the Fregean platonist now has to acknowledge, it seems, that even if we had never arrived at the concept of number, still less done any number theory, but had utilized for our arithmetical purposes only the vocabulary of second-order predicate logic with identity, we should nevertheless have been referring unwittingly to numbers. And the idea of such a community-wide unwitting reference seems to be in tension with the very notion of reference.

Unintentional reference is, of course, a genuine occurrence, as when a man at a fancy-dress party unintentionally refers to his neighbour by use of 'that fool dressed up as Mr Punch'. But here the unintentional reference is the by-product of an intended reference to the very same object. A different case is where someone misunderstands a singular term but is nevertheless said to have referred, via use of it, to its proper designation —when, as we say, they do not know whom or what they are speaking about. Such a case might even be one where the speaker had no concept of what they were speaking about—I mean, we might describe it like that. But even that is no analogy for what the platonist needs. If a community speaks only in terms of the vocabulary of the right-hand sides of the direction equivalences, for instance, and is quite innocent of the concept of direction, there need be no expression in use which they do not fully understand. Unintentional reference is a phenomenon of factual misapprehension or imperfect understanding. Neither rubric covers the present kind of case. The platonist seems to be committed to the bewildering claim that one can fully understand a class of contexts by which references of a certain kind are effected, yet be unaware that any such references are effected and have no concept of the kind of thing to which reference is being made.

A platonist might brazenly try to insist that the right-hand statements are indeed only imperfectly understood by subjects who have not grasped their equivalence to the left-hand statements. But it would be an unhappy stance; for it would remain that such a 'misunderstanding' of statements about lines and their properties and relations would be quite consistent with an apparently perfect grasp of the vocabulary of these statements and of their syntax. The proper response to the objection is rather, it seems to me, to point out that it confuses reference with ontological commitment. So far, we have been content to speak of the semantic role of singular terms as essentially one of reference, but now it is time to be more careful. Not every use of what is—by the Fregean's syntactic criteria—a singular term is a referring use; though every such use is, of course, existentially committing. The contrast is that which Donnellan aimed to draw between *referential* and *attributive* uses of
p. 166 definite descriptions.[20] The platonist must hold, undeniably, that an ↳ endorsement of appropriate statements about lines, etc., *commits* a subject to the existence of the relevant directions. But that is not the same as saying that they unwittingly refer to the directions to which they are committed. To use a sentence in such a way that a reference is effected to a particular object, it seems to me, is to use a sentence in such a way that a full understanding, in context, of the statement made presupposes identifying thought of that object. Since identifying thought involves bringing the thought-of object under some concept or other—and under the relevant covering sortal in the case of abstract objects—it follows that the objection is quite right: it would be absurd to regard speakers whose arithmetical discourse was restricted to what could be formulated in second-order predicate logic with identity as unwittingly referring to numbers. It is also true that, on any plausible account of the matter, the making of a statement which involves reference to a particular object cannot express the same thought as one which does not. Reference is no eliminable aspect of a thought. And it is clear that the platonist will want to regard at least some uses of the left-hand-side sentences as involving reference, in the strict sense of the term, to abstract objects. But what follows is only that platonism must not identify the thoughts respectively expressed by uses of the sentences on the right-hand sides and left-hand sides of the equivalences. And there would, presumably, be no inclination to do that in any case since the conceptual resources involved in grasping the two kinds of thoughts are evidently different.

What platonism needs to hold is, first, that statements which have the same truth-conditions may express different thoughts; second, that if a statement involves reference to an object of a particular sort, we cannot lay it down as a necessary condition for another statement to have the same truth-conditions that it too involves reference to that object—the most we can say is that it must entail that the object in question exists. The first is uncontroversial; mathematics is especially replete with examples of equivalences where quite different conceptual resources are called on by an understanding of the equivalent statements. Think, for instance, of statements concerning right-angled triangles and their equivalents afforded by Pythagoras' Theorem. As for the second, a proper appraisal would have to engage the issues which Donnellan was addressing when he first drew the distinction between referential and attributive uses of definite descriptive phrases.

Pursuit of the matter now would take us too far afield. But a simple example may provide illustration of the perspective in which the Fregean platonist needs to—and, I believe, can—put the crucial equivalences. Someone who knows that first cousins are children of siblings and that $n + 1$th cousins are children of nth cousins may need time to realize that second cousins share two great-grandparents. So such a person, on being told, by way of introduction, that 'This girl and you are second cousins' may not recognize the equivalence of that to 'The closest ancestors which you and this girl have in common are great-grandparents of you both.' And
p. 167 this equivalence, it is ↳ arguable, is unimpaired, even if the elderly couple in question are known to the

speaker and his use of ‘The closest ancestors which you two have in common’ is referential. But we do not want it to be an implication of that equivalence that such a reference is implicit in the original use of ‘This girl and you are second cousins’. For one thing, the introducer may have no identifying knowledge of the elderly couple, and so not be in a position to refer to them. For another, it is not clear that grasping the relation, ‘is a second cousin of’, involves having the concept of an ancestor or indeed a concept of the ancestral of any relation at all.

5. Conclusion: The Onus of Proof

To revert to Field's discussion. Field draws a distinction between *proving* platonism and merely defending it against certain epistemological objections, and raises the question whether *Frege's Conception* might succeed at the second even if not (as he considers) at the first.[21] His answer, based on the Ontological Argument objection —the ‘mystery’ of objects ‘flowing’ from concepts, etc.—is negative. But the way Field draws the distinction emphasizes the most important feature of his view of the platonist project. Field's complaint is not that *Frege's Conception* fails to make a case for platonism by standards acknowledged in that book. Rather, we are not agreed about what ‘proving’ platonism should involve. For Field it is a matter of showing that a relevant *theory* —encompassing, *inter alia*, any germane equivalences—is true. *Frege's Conception* did not attempt to do that—nothing could do it. For the Fregean platonist about number, or direction, in contrast, a proof would consist in showing that a genuinely sortal concept of number/direction can be formed by stipulating that the appropriate equivalences are to hold true, or shown to be already in place by demonstrating that such equivalences are analytically true, followed by further straightforward moves consequent on verification of appropriate statements on the right-hand sides. *Frege's Conception* contains no proof of this view, because it did not conclusively establish that the relevant notions are genuinely sortal. Establishing that would require seeing off, once and for all, the various objections—based on causalist conceptions of reference, etc.—to the syntactic account of singular termhood, and also meeting certain special objections which are consequent on the second-order character of the concept of number, which involves that the explanations of arithmetical vocabulary can take the form of eliminative paraphrases only in the first instance. What I tried to do in *Frege's Conception* was map out the form which a proper defence of platonism would take, weed out some popular but feeble
p. 168 objections and outline what seemed to me more serious ones, and make at least the beginnings of a case for ↳ thinking that they can be answered. If those objections can be answered, then proving platonism about, for example, number is a *triviality*. But Field's call for proof is not a call for a demonstration that those objections can be met. He makes nothing of them and seems content to accept number as a sortal concept. What he is calling for is a demonstration that it is legitimate, a fortiori intelligible, to treat the equivalences in the fashion followed by *Frege's Conception*—that it is indeed a feature of the concept of number that the right-hand sides of the appropriate equivalences encode a priori necessary and sufficient conditions for the truth of the left-hand sides. If that is accepted, the distinction between proving platonism and defending it against the usual epistemological objections near enough collapses; all that the proof requires, in addition, is the truth of appropriate right-hand statements.

As I stressed in *Frege's Conception*, you cannot force someone to accept a concept. What Field needs to deny is that it is *possible* to establish concepts with the characteristics—a priori truth of the relevant equivalences—which the platonist wants. Such a denial can only be based on disclosure of something unintelligible on the route. But Field does not accomplish that—his criticisms are all based either on presupposition of his own ‘theoretical’ view of the equivalences or on the spurious comparisons (Greek mythology, the Ontological Argument, and so on) which we have reviewed.

Notes

1 See Wright (1983).
2 See e.g. Dummett (1973: 494–8). But Dummett is unhappy with this way of defending platonism. See Wright (1983: 64 ff.) for references and discussion of Dummett's reservations.
3 Hale (1987).
4 Field (1984*b*).
5 Dummett (1973: ch. 4); Hale (1979; 1987: ch. 2).
6 See Wright (1983: ch. 4 *passim*, and 180–4 (n. 8)), for an outline and discussion of some of the principal issues.
7 Frege (1884: § §64–7).
8 A point first made by Alston in his (1958).
9 See n. 2 above.
10 The objections raised by causal theories of knowledge and reference, and Dummett's objections, are discussed in Wright (1983: ch. 2, § §xi, xii, and x respectively). The causality objections receive a sophisticated and effective criticism in Hale (1987: chs. 4, 6, and 7).
11 Field (1984*b*: 651).
12 Ibid. 644.
13 Wright (1983: 14).
14 Field (1984*b*: 646).
15 Wright (1983: 148–52).
16 See Wright (1988).
17 Field (1984*b*: 659 f).
18 Ibid. 660.
19 Wright (1983: 129).
20 Donnellan (1966).
21 Field (1984*b*: 661 f).