

HORSE SENSE

Author(s): BOB HALE and CRISPIN WRIGHT

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## HORSE SENSE

Frege's mature philosophy of logic and language was, famously, susceptible to two paradoxes: the contradiction implicit in Basic Law V of his *Grundgesetze*, and the paradox of apparently definite descriptive phrases like "the concept *horse*" which intuitively ought to stand for concepts but which, on Frege's account, cannot do so. The first has generally, on first notice and ever since, been regarded as lethal for the logicist project as Frege conceived it; the second as a less serious difficulty which it was not absurd for Frege to regard as a "mere awkwardness of language," to be taken *cum grano salis*. In other work, we have argued in detail that the damage done to Frege's epistemology of arithmetic and analysis by the inconsistency of Basic Law V has been greatly overestimated. Here, by contrast, our contention will be that Frege's problem with "the concept *horse*" is no mere awkwardness of language, but a serious difficulty which has its source in principles which are integral to his philosophy and which must be resolved if a broadly Fregean approach to the ontology of pure mathematics is to be sustained. Since we take that approach to be the most attractive extant response to the so-called Benacerraf problem, there is much at stake in the issue whether the paradox can be satisfactorily resolved in a fashion consistent with it.

I. BACKGROUND: FREGE'S ONTOLOGY, HIS RESPONSE TO KERRY,  
AND THE PARADOX OF "THE CONCEPT *HORSE*"

There are, in Frege's view, just two fundamentally different kinds of thing—*objects* and *functions*. Within the category of objects, Frege distinguishes between those, such as the planets, which are actual (*wirklich*) and those, such as numbers, which are not, but which are, he insists, equally real or objective.<sup>1</sup> Functions, in Frege's extended sense,<sup>2</sup> include *concepts* and *relations*, and subdivide into levels according to the nature and level of their arguments, the level of a function

<sup>1</sup> Cf. Gottlob Frege, *Die Grundlagen der Arithmetik* (Breslau, Poland: Wilhelm Koebner, 1884); reprinted with English translation by J. L. Austin as *The Foundations of Arithmetic* (Oxford, UK: Blackwell, 1950), p. 35: "I distinguish what is objective from what is handleable or spatial or actual"; and p. 72: "To give spatial co-ordinates for the number 4 makes no sense; but the only conclusion to be drawn from that is that 4 is not a spatial object, not that it is not an object at all. Not every object has a place."

<sup>2</sup> For which, see Frege, "Function and Concept," in *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black (Oxford, UK: Blackwell, 1952), pp. 21–41.

being one greater than that of its highest-level argument. Thus first-level functions are those that take objects as arguments, while second-level functions take first-level functions as arguments, and so on for higher levels. Concepts and relations are those functions which map objects or functions of lower level to one or other of two particular objects, the truth-values the True and the False—for example, first-level concepts and relations are functions from objects to truth-values.

This division of things into *ontological* categories rests upon Frege's analysis of language—upon what is, for him, a prior division of expressions into *logical* categories. Here, the fundamental distinction is between proper names<sup>3</sup> and sentences, on the one side, and on the other, predicates, relational, and functional expressions of the various types and levels. Proper names and sentences, Frege says, are *complete*, in contrast with the various kinds of functional expression, which are *incomplete* or *unsaturated* (*ungesättigt*).<sup>4</sup> The basic types of expression are proper names and sentences.<sup>5</sup> In terms of these, the various kinds of incomplete expression are defined: first-level predicates are those expressions which can be combined with proper names to form sentences, first-order quantifiers those which can be applied to first-level predicates to form sentences, sentential operators those which form sentences from sentences, and so on. It is in terms of these categories of expression that the various kinds of entity recognized in Frege's ontology are defined.

Expressions of all logical types are treated by Frege as having both a sense and, normally, a reference, or semantic value.<sup>6</sup> Objects are

<sup>3</sup>In a broad sense, under which Frege includes all singular noun phrases which he takes to function as conveying reference to particular objects. We shall often use "proper name," or even just "name," in Frege's broad sense.

<sup>4</sup>Frege explains the sense in which he holds functional expressions to be incomplete in his paper "Function and Concept," cited in note 2—see especially p. 25. See also Harold W. Noonan, "The Concept Horse," in P. F. Strawson and Arindarm Chakrabarti, eds., *Universals, Concepts and Qualities: New Essays on the Meaning of Predicates* (Burlington, VT: Ashgate, 2006), pp. 155–76, see pp. 159–61.

<sup>5</sup>In later work, of course, Frege assimilates the latter to the former—sentences are taken to be a species of (complex) proper name, the object named by a (declarative) sentence being one of the two truth-values. It is worth noting that a form of the concept *horse* paradox attends this move. For Frege also thinks we can *name* the truth-values by means of the expressions "the True" and "the False." The result is a clash with the Reference Principle (see below)—since manifestly we cannot substitute "the True" for any true sentence *salva congruitate*.

<sup>6</sup>Notoriously, Frege acknowledges the possibility of expressions (for example, proper names) which have sense but lack reference. See Frege, "On Sense and Reference," in Geach and Black, eds., *op. cit.*, pp. 56–78, see p. 58. Whether he can coherently do so is controversial—see, for example, Michael Dummett, *Frege: Philosophy of Language* (London, UK: Duckworth, 1973), p. 160ff.; and Gareth Evans, *The Varieties of Reference* (New York: Oxford, 1982), p. 22ff. But we will not pursue the issue here.

then simply what proper names stand for, and functions what incomplete expressions stand for—in particular, concepts and relations are just the semantic values of predicates and relational expressions. Of course, Frege should not be interpreted as acknowledging the existence only of those objects and functions which actually have names—that is, are the referents of some expressions of appropriate type in some actual language. The sense in which objects *are* what proper names stand for is rather that to be an object is to be something for which there *could* be a proper name; likewise, *mutatis mutandis*, for functions.

The distinction between concepts (and more generally, functions) and objects is, in Frege's view, absolute. Just as there can be no expression which is both a proper name and a predicate (or other type of incomplete expression), so there can be no thing which is both an object and a concept. In "On Concept and Object," Frege defends this thesis against Benno Kerry's claim<sup>7</sup> that the properties of being a concept and being an object are not mutually exclusive and that, much as a man may be both a father and a son, so concepts may be objects—for example, when we say that the concept *horse*<sup>8</sup> is a concept easily attained, the concept *horse* appears as an object which, if what we say is true, falls under the concept, *concept easily attained*. Frege's response merits close scrutiny; for it reveals—in the shape of the notorious paradox of the concept *horse*—a serious problem for his theory of logical types of expression and the ontology he bases upon it.

As Harold Noonan observes,<sup>9</sup> Frege's response to Kerry involves two claims: first, that we must recognize a distinction "between what can occur only as an object, and everything else"<sup>10</sup>; and second, that nothing can be both a concept and an object. The first claim, as Noonan notes, is not especially radical—it appears roughly equivalent to the traditional doctrine that particulars (in contrast with universals) may figure only as subjects of predication, never as predicated of other things.<sup>11</sup> But while Frege's claim itself may, for that reason, seem relatively unproblematic, what he says in support of it is less so.

<sup>7</sup>As reported in Frege, "On Concept and Object," in Geach and Black, eds., *op. cit.*, pp. 42–55, see pp. 43, 45.

<sup>8</sup>Except when quoting others, we italicize expressions for concepts.

<sup>9</sup>Noonan, *op. cit.*, p. 161.

<sup>10</sup>Frege, "On Concept and Object," p. 44.

<sup>11</sup>Of course, Frege *is* in disagreement with the part of the traditional doctrine of universals that is implicit in the parenthesis—that is, that universals can occur both as predicated of other things, and as themselves subjects of predication. We return to this point below.

Frege writes:

The concept (as I understand the word) is predicative. On the other hand, a name of an object, a proper name, is quite incapable of being used as a grammatical predicate.<sup>12</sup>

In defense of this last assertion, Frege argues that while it is true enough that one can assert of a thing that it is Alexander the Great, or is the planet Venus, for example, this should not be taken as showing that one can after all use a proper name as a predicate—for these examples involve the “is” of identity, rather than the copula (“is” functioning “as a mere verbal sign of predication”), so that

what is predicated is not *Venus* but *no other than Venus*....We have here a word ‘Venus’ that can never be a proper predicate, although it can form part of a predicate. The reference of this word is thus something that can never occur as a concept, but only as an object....But this would mean admitting a distinction...between what can occur as an object, and everything else.<sup>13</sup>

In drawing his conclusion, Frege passes from a premise about *language*—for example, that “Venus” can never be used as a predicate—to a conclusion about *nonlinguistic entities*—that the word’s *referent*, Venus “can never occur as a concept, but only as an object.” We take his conclusion to be equivalent, near enough, to the claim that no statement can incorporate a predicate (concept-word) having Venus as its referent; rather, if Venus is to be the referent of any expression in a statement, that expression must be a proper name.

The inference is more puzzling than the speed and ease with which Frege makes it might lead one to suspect. For that “Venus” cannot, by itself, serve as a predicate is surely consistent with the possibility that what it refers to should also be capable of being referred to by means of some other expression which *could* so function. Since Frege’s conclusion, as we understand it, denies that Venus could be the referent of a predicate, it seems clear that it cannot follow from his stated premise alone. What, we may ask, is the additional premise on which Frege is here relying, but which he saw no need to make explicit? For the moment, we shall simply mark the need for an answer to this question, and turn to Frege’s defense of his second claim, that no concept can be an object.

Kerry, as we saw, suggests that one may exhibit an example of a concept that is at the same time an object by noting that we can

<sup>12</sup> *Ibid.*, p. 43.

<sup>13</sup> *Ibid.*, p. 44.

significantly assert “the concept ‘horse’ is a concept easily attained.” Frege’s response is brisk:

Quite so; the three words ‘the concept “horse”’ do designate an object, but on that very account they do not designate a concept, as I am using the word. This is in full accord with the criterion I gave—that the singular definite article always indicates an object, whereas the indefinite article accompanies a concept-word.<sup>14</sup>

In other words, Kerry’s (or any other) attempt to produce an example of a concept-object is simply *bound* to fail, because any successful candidate for objecthood must be such that it *can only* be referred to by means of a proper name, while any successful candidate for concepthood must be such that it *cannot* be so referred to—clearly, nothing can satisfy both conditions.

But here again Frege’s response involves an inference, signaled by his words “on that very account,” which calls for comment. Once again, the ease with which he makes the move suggests that he does not see it as calling for much justification. The suggestion carried by his explanatory addition “as I am using the word [‘concept’]” is, perhaps, that it is built into the very use of the terms, as Frege understands them, that any expression that designates an object cannot also designate a concept.<sup>15</sup> But that is not obviously so. It is true, as we have already stressed, that Frege’s ontological classifications are to be understood in terms of a prior division of types of expression—so that by an object Frege means anything that can be referred to by means of a proper name, and that by a concept he means anything that can be referred to by a predicate. But that is clearly insufficient, even together with the further premise that no proper name is a predicate, to rule out the possibility of entities that are referred to both by proper names and by predicates, and so are objects which are also concepts. The minimum supplementary assumption Frege needs to deliver his conclusion—as far as we can see—is that objects can *only* be referred to by means of proper names, or at least, that they cannot be referred to by means of predicates. And this is indeed Frege’s view. As he puts it in summarizing his position a little later in the article:

We may say in brief, taking ‘subject’ and ‘predicate’ in the linguistic sense: A concept is the reference of a predicate; an object is something

<sup>14</sup> *Ibid.*, p. 45.

<sup>15</sup> Frege repeatedly emphasizes that Kerry’s objections rest upon misunderstanding his use of “concept.” In “On Concept and Object,” for example, he writes, “Kerry’s misunderstanding results from his unintentionally confusing his own usage of the word ‘concept’ with mine” (p. 42); “[Kerry] is not taking the word ‘concept’ in my sense, and it is not in what I have laid down that the contradiction lies” (p. 46).

that can never be the whole reference of a predicate, but can be the reference of a subject.<sup>16</sup>

Granted the additional premise suggested, Frege's second inference can be reconstructed as follows. He assumes the following:

- Objects 1:* Something is an object iff it can be referred to by a proper name.  
*Concepts 1:* Something is a concept iff it can be referred to by a predicate.  
*Objects 2:* If something is an object, it can only be referred to by a proper name.  
*Exclusion:* No proper name is a predicate.

And the inference then proceeds:

- |   |                                 |
|---|---------------------------------|
| (1) $\alpha$ is both an object and a concept.                   | assumption, for <i>reductio</i> |
| (2) $\alpha$ can only be referred to by means of a proper name. | (1), <i>Objects 2</i>           |
| (3) $\alpha$ can be referred to by means of a predicate.        | (1), <i>Concepts 1</i>          |
| (4) $\alpha$ cannot be referred to by means of a predicate.     | (2), <i>Exclusion</i>           |
| (5) $\alpha$ is not both an object and a concept.               | <i>reductio</i>                 |

What about Frege's earlier inference, to the conclusion that whatever is referred to by an expression, for example, "Venus," that cannot feature as a predicate cannot be the referent of any predicate? Well, the extra premise Frege needs is an analogue for concepts of *Objects 2*, namely:

- Concepts 2:* If something is a concept, it can only be referred to by a predicate.

We may then reconstruct his inference as follows:

- |  |                                 |
|--|---------------------------------|
| (1) "Venus" cannot serve as a predicate.                           | assumption                      |
| (2) The referent of "Venus" can be the referent of a predicate.    | assumption, for <i>reductio</i> |
| (3) The referent of "Venus" is a concept.                          | (2), <i>Concepts 1</i>          |
| (4) The referent of "Venus" can be referred to by "Venus."         | obvious                         |
| (5) "Venus" is/can serve as a predicate.                           | (4), <i>Concepts 2</i>          |
| (6) The referent of "Venus" cannot be the referent of a predicate. | <i>reductio</i>                 |

These reconstructions seem to us pretty much inescapable. But they immediately leave us with a further question: why did Frege endorse

<sup>16</sup> *Ibid.*, pp. 47–48.

the extra principles we have called *Objects 2* and *Concepts 2*? This will be the matter for the next section.

Before that, we should remind the reader that Frege's brisk response to Kerry lands him, equally briskly, in a paradox: for he commits himself to claiming that the concept *horse* is not a concept but an object—in direct conflict with the plausible assumption, made by Kerry, that if “the concept *horse*” refers to anything, what it refers to is a concept. Indeed, the assumption is not merely plausible—it is one to which, on the face of things, Frege is himself committed, by his own principles. For he holds that the reference of an expression is determined by its sense. But “the concept *horse*” appears to be formed from a functional expression “the concept...” which, by virtue of its sense, stands for a function whose value, for a given argument, is a certain concept—from which it seems to follow that, if “the concept *horse*” refers at all, it *must* stand for a concept.

Frege, of course, is fully appreciative of the problem. His first response to it—in his 1892 paper—is well known, and culminates in his notorious plea for a *granum salis*:

It must indeed be recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept *horse* is not a concept, whereas the city of Berlin is a city, and the volcano Vesuvius is a volcano....In logical discussions one quite often needs to assert something about a concept, and to express this in the form usual for such assertions—viz. to make what is asserted of the concept into the content of a grammatical predicate. Consequently, one would expect that the reference of the grammatical subject would be the concept; but the concept as such cannot play this part, in view of its predicative nature; it must first be converted into an object, or, speaking more precisely, represented by an object.<sup>17</sup>

I must admit that there is a quite peculiar obstacle in the way of understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway—who does not begrudge a pinch of salt.<sup>18</sup>

<sup>17</sup> *Ibid.*, p. 46.

<sup>18</sup> *Ibid.*, p. 54.

However, sometime after the publication of “On Concept and Object”—certainly by 1906, and probably much earlier<sup>19</sup>—Frege abandoned this response. And he was surely right to be dissatisfied with it. The problem goes far deeper than a mere “awkwardness of language.” If Frege is right in his denial that singular reference to concepts, and more generally functions, is even so much as possible, then it is, to put it mildly, seriously open to doubt whether his whole conception of logico-linguistic types, and the ontology based upon it, can even be coherently stated. For the articulation of his theory, it seems, requires us to speak, at least in general terms, of concepts, relations, and functions. We need to say, for example, that predicates, relational, and functional expressions, of the various levels, have concepts (or properties), relations, and functions, as their semantic values; that functions of any given level take only functions of lower level, or objects, as their arguments; and, of course, that no concept, relation, or function can be the semantic value of any proper name. Taken at face value, such statements involve the use of “concept,” “relation,” and “function” as general sortal terms, and deploy first-order quantification, with associated pronominal cross-reference, restricted by means of them. Yet where first-order quantification, associated pronominal cross-reference, and general sortal terms are allowed as legitimate and intelligible, it seems compelling that singular reference to the elements of the relevant domain must be in equally good standing—contrary to what the theory itself claims! So the very coherence of Frege’s theory is in doubt.

Whether the paradox really is symptomatic of a deeper incoherence in Frege’s theory, so that at least one of its key claims must be rejected or revised, or whether, rather, the problem is merely superficial, and can be dealt with whilst leaving the theory itself essentially unchanged, are questions to which we shall very shortly turn. First, we need to elicit and comment briefly upon the underlying general principle which appears to lie at the heart of the difficulty.

<sup>19</sup> In his plan for a critique of Schoenflies’s *Die logischen Paradoxien der Mengenlehre*, dated 1906, published in his *Nachlass* (*Posthumous Writings*, ed. Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach, trans. Peter Long and Roger White (Chicago: University Press, 1980)), Frege identifies the source of the trouble as being the use of such expressions as “the concept *horse*” and the word “concept” itself, which he now regards as defective. According to Michael Dummett, Frege had already seen this, and proposed a new solution to the problem, soon after the publication of “Über Begriff und Gegenstand” in the *Vierteljahrsschrift für wissenschaftliche Philosophie* in 1892. See Dummett, *op. cit.*, p. 212.

## II. THE REFERENCE PRINCIPLE

According to our analysis, Frege's response to Kerry relies upon some general principles which are not made fully explicit in "On Concept and Object," but which are either implicit in Frege's text or at least strongly suggested by things he does explicitly assert, or by inferences he explicitly draws. These were:

- Objects 1:* Something is an object iff it can be referred to by a proper name.
- Concepts 1:* Something is a concept iff it can be referred to by a predicate.
- Objects 2:* If something is an object, it can only be referred to by a proper name.
- Concepts 2:* If something is a concept, it can only be referred to by a predicate.

Amalgamating these principles in pairs suggests the following respective conceptions of objects and concepts:

- Objects:* An object is anything which can, and can only, be referred to by a proper name.
- Concepts:* A concept is anything which can, and can only, be referred to by a predicate.

These in turn are instances of a general schematic principle plausibly taken as underpinning Frege's whole approach to ontology:

An entity of a certain *kind* is anything which *can, and can only*, be referred to by an expression of a certain correlative logico-syntactic *type*.

Given that the relevant *types* of expression are all mutually exclusive, just as proper names and predicates are, it will follow that the corresponding kinds of entity are likewise mutually exclusive—as befits the idea that they constitute basic *categories*.

Suppose now that two expressions  $\xi$  and  $\zeta$  refer to the same thing. What is referred to will belong to a certain kind, or category. As such, it will be something which can be referred to by, and only by, expressions of a certain syntactic type. So  $\xi$  and  $\zeta$  must be expressions of the same type. Hence, if we are right, Frege's view in general is that two expressions co-refer only if they are of the same syntactic type. Since the criterion for expressions to be of the same syntactic type is that they can be interchanged in all contexts preserving well-formedness, or *salva congruitate*, in one sense of that term, the salient corollary is that:

Two expressions co-refer only if they are everywhere interchangeable *salva congruitate*.

Although the claim cannot, so far as we are aware, be supported by explicit quotation, we conclude that there can be little doubt that Frege

embraces both the general schematic principle above—we will call it the *Reference Principle*—and its corollary.<sup>20</sup> This interpretation both best explains his willingness to infer directly from the premise that the words, “the concept *horse*,” designate an object, to the conclusion that “on that very account they do not designate a concept” and provides the simplest, most natural theoretical setting for Frege’s various informal explanations of the ontological categories of object and concept, or more generally function. The thesis that expressions of different logical types cannot co-refer—equivalently, that if two expressions are to share their reference, they must be at least interchangeable *salva congruitate*—thus appears to be a lynchpin of Frege’s ontology.

Since it plays a leading role in generating the paradox, the Reference Principle must be a prime candidate for revision, should it prove necessary to revise Frege’s theory to escape the paradox. But is any revision actually needed? We must consider first whether there may be any effective response to the problem which is consistent with retention of Frege’s theory as we have outlined it.

### III. BLIND ALLEYS

Perhaps the most obvious response consistent with retention of Frege’s theory as it stands—including, crucially, its denial of the apparent possibility of singular reference to concepts, relations, and functions—would be to argue that its resort to sortal terms for, and quantification over, concepts, relations, and so on, is a mere convenience, making possible a simpler and more concise presentation of the theory, but dispensable in principle in favor of a strict formulation which eschews all such devices.

Alternatively, it might be suggested that while such devices cannot be fully eliminated by means of any such systematic reductive paraphrase, they are not what they seem. We have indeed the *appearance* of general sortal talk about concepts, relations, and so on, and of first-order quantification over such things, but the appearance is not to be trusted. It no more involves genuine reference to concepts, relations, and so on, than talk of actions done for the sake of so-and-so, or such-and-such, involves genuine reference to a mysterious new kind of entity, *sakes*.

<sup>20</sup> In an earlier paper, Crispin Wright uses “the Reference Principle” as a label for the corollary—see “Why Frege Does Not Deserve His Grain of Salt: A Note on the Paradox of ‘The Concept Horse’ and the Ascription of *Bedeutungen* to Predicates,” in Johannes L. Brandl and Peter Sullivan, eds., *New Essays on the Philosophy of Michael Dummett, Grazer Philosophische Studien 55* (Atlanta: Rodopi, 1998). We refer to this paper as “Wright 1998” but cite page numbers from its reprinting in Bob Hale and Crispin Wright, *The Reason’s Proper Study: Essays towards a Neo-Fregean Philosophy of Mathematics* (New York: Oxford, 2001), pp. 72–90.

We think neither of these responses holds any promise at all. After explaining why, we shall conclude this section by reviewing and rejecting a third attempt to rescue Frege's position, by appeal to the idea that the paradox is the product of a misguided effort to say what can only be *shown*.

*Blind Alley 1: Eliminative Paraphrase.* Frege's theory can hardly be stated without *somehow* talking in general terms about the semantic values (concepts, relations, and more generally, functions) of incomplete expressions. The first course therefore requires a general method of paraphrasing out all uses of such apparent sortal terms as "concept," "relation," and "function," and apparent uses of first-order quantification over concepts, relations, and functions, together with any associated pronominal cross-reference. Everything the theory says, deploying such devices—not only claims about specific concepts, relations, and functions, but also, and crucially, general claims—must be expressed without their use, in such a way that all reference to incomplete entities, whether definite or indefinite, is carried by incomplete expressions.

What are the prospects? There are three main kinds of talk of concepts, relations, and functions generally, which require treatment—(i) apparent *singular reference* to concepts, and so on, (ii) uses of ostensibly *sortal predicates* formed with the general terms "concept," "relation," and so on, and (iii) general statements involving apparent *first-order quantification* over concepts, and so on. Since, further, any adequate statement of the theory must involve claims about the kinds of semantic value associated with the various different kinds of expression, and since such claims ostensibly involve definite or indefinite reference to concepts, and so on, of the proscribed sorts, we shall need also to see how such claims may be strictly re-expressed.

There are, no doubt, some easy cases. For example, statements featuring ostensibly singular reference to concepts, relations, and so on, such as:

The concept *wise* is a concept easily attained.  
The relation of brotherhood is nonsymmetric.

are readily paraphrased by such as:

It is easy to learn what is required for it to be true of someone that she is wise.

One person may be brother to another without the latter being brother to the former.

But more work is required to eliminate uses of such apparent first-level sortal predicates as "...is a concept," "...is a relation," "...is a

function of two arguments,” and the like, even in such relatively simple statements as:

The concept for which “ $x$  is wise” stands is a first-level concept.  
 Temporal precedence is a binary relation.  
 The function  $x^3 + y$  is a first-level function of two arguments.

As Michael Dummett points out,<sup>21</sup> since first-level concepts cannot properly be referred to by names, but are what first-level predicates stand for, claims about them must be of at least second-level. Thus what is required here is to see how to dispense with such apparent (but in Frege’s view specious) first-level predicates as “...is a first-level concept” by genuine second-level predicates. Dummett’s own proposal starts from the observation that an expression such as “what ‘ $\xi$  is wise’ stands for” can function as what he calls a “predicative expression,” in which it stands for, and can be used to ascribe, a property—in contrast with, say “what Eve gave Adam” in its most likely use, in which it is a singular term, meaning the object Eve gave Adam. Thus “Solomon was what ‘ $\xi$  is wise’ stands for” can be understood as equivalent to “Solomon was wise.” What is required, of the needed second-level predicate, is that it should be true of all (and only) first-level concepts. Dummett proposes “...is something which everything either is or is not,” where “something” is to be understood as expressing second-level generality, and “everything” first-level. Thus we can employ:

What “ $\xi$  is wise” stands for is something which everything either is or is not.

to state clearly, so Dummett suggests, what we mean, but cannot properly express, by saying that the concept *wise* is a first-level concept.<sup>22</sup>

One difficulty with this proposal as it stands is that it represents the *analysandum*—the statement that something is a first-level concept—as building in an endorsement of the generalized law of excluded middle. While Frege would hardly have objected, given his insistence that concepts must have sharp boundaries, it would be a cause for concern for anyone who thinks, to the contrary, that we should not exclude the possibility of vague concepts, or other failures of bivalence. But this is a minor irritant—the difficulty could be avoided by an alternative choice of second-level predicate, such as “...is something which nothing is or something could be.”<sup>23</sup> A more serious problem is that Dummett’s claim that predicative expressions share

<sup>21</sup> Dummett, *op. cit.*, pp. 215–17.

<sup>22</sup> *Ibid.*, p. 216.

<sup>23</sup> This difficulty was first noted, and this remedy proposed, in Wright 1998, see pp. 78–79.

their referents with corresponding predicates—so that “what ‘ $\xi$  is wise’ stands for” stands for the same thing as “ $\xi$  is wise”—clashes directly with the Reference Principle. For the result of substituting the former for the latter in “Socrates is wise,” for example, is “Socrates what ‘ $\xi$  is wise’ stands for,” and this is simply ill formed.<sup>24</sup>

Dummett concedes that “the terminology that would be required for speaking, in a logically correct manner, about the referents of predicates and relational expressions is...cumbersome and verbose.”<sup>25</sup> But, even supposing the foregoing objection can be surmounted,<sup>26</sup> there are other and more difficult examples which strongly suggest obstacles of principle to carrying through the requisite linguistic reforms. New problems are posed even by such simple-seeming statements as:

First-level predicates stand for concepts.

To deal with this, we need to quantify over expressions. In itself, this is no problem—expressions are a kind of object, so there is no reason why we should not use ordinary first-order quantification restricted by means of a predicate “is a first-level predicate.” Thus a partial rendering of our target statement is:

$\forall x(x \text{ is a first-level predicate} \supset x \text{ stands for a concept})$

But how are we to deal with the consequent? It is naturally taken to have the structure

$\exists y(x \text{ stands for } y \wedge y \text{ is a concept})$

This cannot be reckoned satisfactory as it stands, however, because it deploys the proscribed pseudo-predicate “is a concept.” But if we try to eliminate this by Dummett’s method, we are stymied—the result is:

$\exists y(x \text{ stands for } y \wedge y \text{ is something everything either is or is not})$

<sup>24</sup> This objection is pressed in Wright 1998, see p. 80.

<sup>25</sup> Dummett, *op. cit.*, p. 217.

<sup>26</sup> Dummett might, for example, be expected to respond that the need for a copula to precede such terms as “what ‘ $\xi$  is wise’ stands for” if we are to obtain a sentence by combining them with proper names is merely grammatical, and of no logical significance. But as Wright counters (in his 1998 paper, p. 81), to take this line is to completely undermine Frege’s explanation of the incompleteness of predicates and other functional expressions, which consists in their capacity to yield complete sentences when their gaps are filled with proper names. For their possession of that capacity precisely depends upon the presence of the copula, or other similarly functioning devices, such as finite verb endings.

and the trouble with this is all too obvious: the argument to Dummett's proposed second-level replacement<sup>27</sup> for "is a concept" needs to be a predicative expression, or a variable of appropriate type. But the bound variable  $y$  is an *individual-* or *object-*variable, as it must be if it is to be suitable to occupy the second argument place in the predicate " $\xi$  stands for  $\zeta$ ." Thus our attempted paraphrase is simply ill formed.

Equally serious obstacles obstruct any attempt to provide clean reconstructions of such claims as:

First-level functions take only objects as their arguments,

but since the difficulties already exposed seem enough by themselves to show that the eliminative paraphrase strategy is unlikely to succeed, we leave it to the reader to explore the details.

*Blind Alley 2: Pseudo-Reference.* There is no clear prospect, then, of satisfactory systematic paraphrase of the "first-orderish" locutions—apparent singular reference, apparent first-order quantification, and recourse to apparently sortal predicates—that seem indispensable for the statement of a distinctively Fregean semantic theory. The very articulation of the theory seems to call for expressive devices whose semantic function is, *prima facie*, in conflict with the content of the theory, and no means is evident for a systematic explanation of how, *secunda facie*, such devices are not really called for.

But does that *have* to be explained? Might the Fregean not, instead, rest content with an insistence that these devices are not what they seem, that the first-orderish appearances notwithstanding, they do not, semantically, work in a first-orderish way, even though no systematic paraphrase is to be found which explains the appearances away and displays what, if any, semantic role the locutions in question really do have?

Now—invoking once again the rather tired Quinean example—it does indeed appear too strong a demand that a reductive paraphrase be supplied of locutions such as "I am here on *the plaintiff's behalf*" and "The law was changed for *the sake of all those who had suffered under the previous, punitive tax arrangements*" before we can reasonably discount the suggestion that the first-orderish behavior of "sake" and "behalf" is misleading. English does have devices whereby the use of nominal phrases of the italicized kind can indeed be paraphrased away—for instance, "I am here to represent the plaintiff" and "The law was changed to enable those who had suffered under the previous tax arrangements to be treated more fairly"—but even if it did not, there would be no sound motive to treat the apparently definite

<sup>27</sup> Obviously the same goes for Wright's alternative. See note 23.

descriptive italicized phrases as purporting first-order reference. So why should the occurrence of first-orderish devices in Frege's own—and any foreseeable—statement of his theory be taken any more seriously?

There are two things amiss with this suggestion. First, it is obviously unavailable to Frege. The “awkwardness” with “the concept *horse*” arises only and purely because Frege *does* take its apparent singular-referential purport seriously. The suggestion is that we try to mitigate what would otherwise be the through-and-through infelicity of his statement of his theory by denying that the first-orderish devices with which it is littered really have that kind of semantic role. But if we are prepared to do that, we can simply deny that there is any singularity in the first place. We can deny that “the concept *horse*” is singular-referential. There is then no pressure whatever to grant the truth of the paradoxical sentence.

Frege feels under pressure precisely because he wants, and needs, to take the first-orderish appearances at face value. Such appearances are matched in spades by the ordinary discourse of arithmetic and, as is familiar, play a pivotal role in Frege's thinking about the ontology of arithmetic. *Fregean platonism* is the very watchword for the idea that a face-value construal of the syntax of number theory, provided the language is then systematically and rigorously accounted for, may precede and underwrite a conception of its subject matter as consisting in a special domain of objects. The presently envisaged form of response to his problem with “the concept *horse*,” absent any further principled distinction between when discourses with *prima facie* first-orderish syntax should be taken seriously and when they should not, would completely cut the ground from underneath this most distinctive and important Fregean line of thought.

It is true, of course, that we, rightly, do not take seriously the singular-referential form of *prima facie* definite descriptive phrases embedding “sake” and “behalf.” The distinction we are implicitly drawing in not taking them seriously—this is the second advertised “thing amiss”—is presumably grounded in the consideration that their first-orderish behavior does not extend very far. We do not, for example, count them, comfortably quantify over them, or construct sentences involving anaphoric reference to them, nor do we have any very developed vocabulary in terms of which to express their mutual similarities and dissimilarities. Perhaps most important of all, we have no clear notion of identity of the sakes and behalves.<sup>28</sup> By contrast, all these devices and locutions

<sup>28</sup> More carefully, we have no notion of the identity conditions of sakes and behalves conceived as distinctive objects. Someone might reasonably say that the sake of  $x$  = the

are everyday fare in arithmetical discourse. And—this is the crux—natural analogues of each of them will occur, or be suggested by, any developed statement of Frege's semantic theory, of the kind prefigured in the first two sections of this essay.

*Blind Alley 3: Saying and Showing.* In a recent article, Harold Noonan advances the view that Frege's paradox is actually unavoidable.<sup>29</sup> In Noonan's view, Frege was right to reject his own initial suggestion that expressions such as "the concept *horse*" and "the function  $2.x^3 + x$ " refer to objects which intrude whenever we wish to speak of the functions in question. Rather, such expressions, along with the pseudo-sortal terms "concept" and "function" themselves, are one and all *defective*. They cannot stand for what they purport to stand for, so they can stand for nothing. The predicates "is a concept" and "is a function," which by their grammar can only be first-level, can only be understood as standing for self-contradictory concepts. Nevertheless, the distinction between objects and functions, including concepts, is "founded deep in the nature of things." There remains an unbridgeable gap between objects, which are saturated entities, and functions, which are unsaturated. The conclusion to which we are driven, Noonan thinks, is the conclusion drawn by Peter Geach in an earlier article, namely, that what Frege requires to address the problem is the distinction made by Wittgenstein in his *Tractatus*, between what can be *said*, and what can only be *shown*.<sup>30</sup>

Of course, whether Wittgenstein succeeded in *drawing* such a distinction, or merely postulated one, is moot. According to Geach, Frege held, and "his philosophy of logic would oblige him to hold," that there are "logical category-distinctions which will clearly show themselves in a well-constructed formalized language, but which cannot properly be asserted in language." Further, while "the sentences

sake of  $y$  iff  $x = y$ , and similarly for behaves. But if sakes and behaves have the same identity conditions as the people whose sakes or behaves they are, how exactly do they differ from those people? Why doesn't "for the sake of  $x$ " just collapse into "for  $x$ "? (As indeed it seems to do.)

<sup>29</sup>Noonan, *op. cit.*, pp. 155–56, 165, and 171.

<sup>30</sup>Our statement of Noonan's view lightly paraphrases the first two paragraphs on p. 165 of his paper. It is true that Noonan does not there explicitly say that the paradox is unavoidable, only that it is "not easily dismissed," but his final conclusion (p. 171) is that it cannot be avoided. At p. 165, Noonan is stating Frege's view, but it is clear that he means to endorse it. If he did not, he could hardly draw the conclusion that the saying/showing distinction provides the only way out. The article by Geach to which he refers is "Saying and Showing in Frege and Wittgenstein," in K. J. Hintikka, ed., *Essays on Wittgenstein in honour of G. H. von Wright, Acta Philosophica Fennica 28* (Amsterdam, Netherlands: North Holland, 1976), pp. 54–70.

in which we seek to convey them in the vernacular are logically improper and admit of no translation into well-formed formulas of symbolic logic," there is all the same "a test for these sentences" having conveyed the intended distinctions—namely, that by their aid mastery of the formalized language is attainable."<sup>31</sup>

What does Geach have to offer in support these claims? After setting out a version of the concept *horse* problem, he writes:

...Frege could and did find a way out. The result of inserting an English expression in the blank between the quotes in the context:

what ' ' stands for

will stand for, *bedeuten*, whatever that very English expression stands for....Thus, the expression:

What 'the Duke of Wellington' stands for

is merely a long-winded substitute for the Duke's name; but on the contrary in the sentence

He is what 'un coquin' stands for in French

the predicate stands for what this French expression does stand for, and the sentence is a long-winded way of saying 'He is a scoundrel'. Similarly

that function of 2 which 'the square of' stands for

is a long-winded way of saying 'the square of 2': it would be nonsense, Frege came to see, to try to use the expression...:

that function which 'the square of' stands for

on its own as an *Eigenname*, or to try to use ["The reference of the predicate '— killed Caesar'"] as an *Eigenname* either.

Frege thus escapes checkmate at this move...<sup>32</sup>

It is not entirely clear exactly what problem Geach supposes that this point enables Frege to elude, even if only temporarily. Perhaps what he has in mind is that in view of the principle stated at the start of the quoted passage, Frege can use expressions of the form: "what ' ' stands for" to refer to whatever the expression that goes in the blank stands for—whether that expression is a proper name (*Eigenname*) or predicate; and that somehow this form of semantic ascent provides the means of direct reference in cases where that would otherwise

<sup>31</sup> Geach, *op. cit.*, p. 55. Geach actually advances four composite theses, from which we have extracted just the claims that matter here.

<sup>32</sup> *Ibid.*, pp. 56–57.

be problematic. But it is unclear how it might be supposed to do so.<sup>33</sup> It does not matter since Geach is anyway confident that this merely postpones check:

...but at the next move there will be a new check not so easily escaped. Frege holds that there is a fundamental difference between concepts and objects. Let us take an instance of this:

(c) There is a difference between what 'Brutus' stands for and what the predicate '— killed Caesar' stands for.

By the principle laid down above, any phrase of the structure:

what ' ' stands for

is replaceable syntactically by simple use of the English expression standing between the quotes—syntactically and indeed *salva veritate*. Let us try to do this with (c)! What we get is:

(d) There is a difference between Brutus and killed Caesar.

And this is manifest nonsense...

Is Frege then checkmated after all? I do not think so. What he must say, and I think would say, is to this effect: 'The reduction of (c) to (d) shows that we cannot really construct any significant proposition to say the sort of thing we were trying to say in (c). And this is not due to some removable defect of ordinary language: in a proper symbolic language, such as my *Begriffsschrift*, an inequality with an *Eigenname* on one side and a predicate or functor standing alone without its argument on the other side would equally not convey what we wish. All the same, sentences like (c) are didactically useful: they may lead someone to understand my *Begriffsschrift*. And the test of his having actually mastered

<sup>33</sup> After all, "...what 'un coquin' stands for in French," used as above, is just another, albeit a metalinguistic, *predicate*; what can it accomplish for us, expressively, that "...is a rascal" cannot? Perhaps Geach had in mind something similar to the confused maneuver with "predicative expressions" suggested by Dummett and rejected above. Noonan, who largely follows Geach, has a more explicit suggestion. We may, he thinks, "say of the concept horse, for example, that it is a concept," but avoid the paradox, by using a perfectly genuine second-level predicate, much as Dummett proposes. In fact, Noonan does not use Dummett's "What 'ξ is wise' stands for is something which everything either is or is not"; instead, he exploits the fact that in Frege's mature theory, sentences are proper names of truth-values, so that we can express the problematic claim:  $\forall x(x \text{ is a horse} = \text{the True or } x \text{ is a horse} = \text{the False})$ . This is a singularly unfortunate choice, since Frege's use of the expressions "the True" and "the False" to stand for what true or false sentences stand for is itself ruled out by the Reference Principle (which Noonan endorses). But for present purposes, the important point is that Noonan agrees with Geach that even if some claims featuring such pseudo-proper names as "the concept horse" and such pseudo-predicates as "is a concept" can be restated in a logically clean way, there must remain others whose intended content cannot be said, but only shown.

the symbolic language is his successful use of it, not his ability to parrot sentences like (c) or produce similar ones on his own account.<sup>34</sup>

As best we can understand it, the reply Geach is proposing on Frege's behalf comes down to four claims: (1) Any attempt to refer by means of proper names to functions simply results in nonsense. (2) There is a fundamental difference between objects and concepts/functions, but we cannot assert this, either by means of sentences like (c) or by any others (including this one!). (3) "The Frege-Wittgenstein notion of what comes out but cannot be asserted is almost irresistible, in spite of its paradoxical nature, when we reflect upon logic."<sup>35</sup> (4) We can however use such sentences as a (strictly nonsensical) heuristic to teach someone to speak a logically correct language, such as *Begriffsschrift*.<sup>36</sup>

So far as we have been able to see, Geach does almost nothing to substantiate claim (3) beyond what we have quoted above—the claim that appeal to what can only be shown is the *only* way out of Frege's fix. He does criticize what he claims to be a widely held view that Frege's problem results from "trying to discuss in the object language what ought properly to be discussed in a metalanguage" and that it can be avoided by semantic ascent.<sup>37</sup> He does not consider the possibility of rejecting or revising the Reference Principle, but pretty clearly simply takes the Principle for granted. More importantly, it seems to us, he provides nothing, beyond the suggestion about the pedagogic utility of abortive attempts to say what can only be shown, to support the claim that his is an *effective* solution to the problem. It seems that, at the very best, the proposed way out will secure a legitimate "didactic" use for a *severely limited range* of strictly ill-formed sentences—near enough, those which record distinctions of category among the referents of particular expressions, such as:

What "Brutus" stands for is an object, whereas what "smote Caesar" stands for is a concept.

But this massively undershoots. What about all those sentences which are supposed to articulate Frege's theory of language, including its semantics and the associated ontology—sentences like:

Proper names stand for objects.

<sup>34</sup> Our extended quotation is from Geach, *op. cit.*, pp. 57–58.

<sup>35</sup> *Ibid.*, p. 56.

<sup>36</sup> As far as we know, Frege never himself makes anything like the final claim; nor does Geach claim otherwise. His attribution of the other claims to Frege is presumably based upon Frege's plan for a critique of Schoenflies, dated 1906 in the *Nachlass*.

<sup>37</sup> *Ibid.*, pp. 58–61. Although Geach says this solution is often offered, he cites no proponents of it, and we can think of no one who has seriously advocated it.

Predicates stand for concepts.

No object is a concept.

First-level functions can take only objects as arguments.

Second-level functions take first-level functions as arguments.

First-level predicates stand for functions from objects to truth-values.

The level of a function is one higher than that of its highest-level argument.

...and so on?

We do not see how any theorist could expect to be taken seriously who maintained that sentences such as these earn their keep as merely didactically helpful devices, that they are strictly ill-formed nonsense and can be assigned no coherently stateable content, even though something coherent, but unstateable, “comes out.” What they—or their nineteenth-century German counterparts—are intended to do is to convey a theoretical account of a logically correct language and its relation to the world. But, according to Geach’s proposal, they can at best assist us in learning such a language. For about how such a language relates to the world, nothing coherent *can* be said. In effect, the proposal enjoins total abstinence from any theorizing of precisely the kind characteristic of Frege’s philosophy of logic and language. It is not just that we cannot *say* what Frege manifestly attempted to say—*there are* no coherent, systematic propositional thoughts to be had about the language-world relationship, *a fortiori* none expressed by the range of sentences in question. But that is tantamount to the acknowledgement that Frege’s semantic theory *is* incoherent—the play with the idea, to the contrary, that there is a coherent something-or-other, some form of nonpropositional insight, which somehow “comes out” but cannot be explicitly formulated, is an arrant, addled fiction.

#### IV. (ALMOST) EVERYMAN’S MISTAKE

We should conclude that the paradox of the concept *horse*, so far from being something that one might swallow with a grain of salt or two, betrays a fundamental incoherence in Frege’s philosophical semantics and associated ontology. There is no living with it.

What has gone wrong? Well, it is the Reference Principle that forces us to say that no two expressions of distinct syntactic category—distinct congruity profile—can co-refer, and hence that there is no referring to what a predicate, *F*, refers to by any kind of singular term, not even by a term of the form, “what *F* refers to.” If there is a solution to be found, then it seems it must involve abandoning or somehow radically qualifying the Reference Principle.

But how? Well, as observed by Wright 1998, there is a long history of confusion in the understanding of the paradox—an oversight of

which, so far as we are aware, all earlier commentators, including Frege himself, were guilty—and this requires disinfection before it can be clear exactly what the range of options for qualifying the Principle comprises. Consider any pair of sentences of the following two forms:

- (1) “*a*” refers to ...
- (2) “*F*” refers to —

where “*a*” is a singular term and what replaces the dots is an expression standing for an object; and “*F*” is a predicate and what replaces the dash is...well, what? Let us not try to say, but simply specify that it is to be an expression which is syntactically fit to indicate (we cannot help but put it this way) *the reference of a predicate*. Then, by the Reference Principle, what replaces the dash has to be itself a predicative expression. But, obviously enough, no such expression will allow of syntactically congruous substitution into the place marked by the dash in (2). The grammar of “refers to” will not permit it. And this, for a Fregean, cannot be just a point about the *surface* grammar of “refers to.” Rather, “refers to” has its own syntactic character, determined by its congruity profile: it requires completion on either side by singular terms. Reference, conceived as the *Bedeutung* of “refers to,” is an object-object relation. And no object-object relation can include concepts in its field. In the sense of “refers to” in which singular terms refer to objects, predicates, and incomplete expressions generally, cannot, for Frege, *refer to anything*.

So we need to rewrite (2) if it is to be apt to convey the reference—better, the semantic value—of the predicate *F*. We need to replace “refers to” by an expression for an object-concept relation. An example would be the relation denoted by the emboldened part of:

- (2)\* “*F*” is **satisfied by an object just in case that object** is *F*,

although this has the drawback of being available to a theorist who denies any kind of *Bedeutung* to predicates.

Let us step back. The Reference Principle actually combines two independent ingredient principles. One we may call the *Single Relation* principle: that some one reference relation uniformly connects expressions of each syntactic type with the kinds of entity that provide their respective semantic values. Reference is reference, the same thing across the board. The other ingredient we may term the *Type-Kind Uniqueness* principle: that *syntactic types* of expression correlate one-to-one with the *ontological kinds* of entity among which their tokens are eligible to refer, so that singular terms are eligible to refer to and only to objects, predicates are eligible to refer to and only to concepts, and so on. This principle, taken with the first, forces us to deny that

any expression of any given syntactic type can co-refer with any expression of a different syntactic type, and hence to deny that “the concept *horse*” can take its reference from within the domain in which predicates refer—the domain of concepts. So we seem required to deny that “the concept *horse*” can refer to any concept, and hence—if we grant it any kind of reference—to affirm that the concept *horse* is not a concept.

But this is a muddle, not a paradox. It is a muddle because the Single Relation principle and the Type-Kind Uniqueness principle are mutually inconsistent anyway. Their inconsistency emerges when the latter is applied to the putative expression of the reference relation itself, which is then forced to fragment into differing syntactic types, in tandem with the variety of syntactic types of expression which may supply its right-hand side, so to speak. These different types of expression for reference are then in turn constrained, by Type-Kind Uniqueness, to pick out relations of different kinds. And this enforced variety of reference relations runs contrary to the Single Relation principle.

Something has to be given up, or somehow qualified. But there are options. A common proposal, before and after Frege, has been to, in effect, reject the Type-Kind Uniqueness principle. For a theorist who adopts this line, both “...is a horse” and “the concept *horse*” may refer—may stand in the same reference relation—to the concept *horse*.<sup>38</sup> This of course was exactly the type of view that was thought to run into trouble over the issue of the unity of the proposition; and over which Frege’s view has been held to constitute an advance. We will come back to that matter shortly.

The alternative is to drop the Single Relation principle. Here is a glimpse of what that will involve. Now, there will be no single semantic relation of *Bedeutung*, linking each of terms, predicates, relational expressions, quantifiers, and functors to their respective types of semantic value. Rather, the varying congruity profiles of these different kinds of expression will go in tandem with a correspondingly varying range of reference relations, each marked by an expression whose congruity profile reflects that of the expressions which may congruously feature, so to say, on its right-hand side, and thereby reflects the type of entity concerned. Each of these reference relations,

<sup>38</sup>This is in effect the view taken (though not in explicit reaction to the concept *horse* paradox) in Strawson, *Subject and Predicate in Logic and Grammar* (London, UK: Methuen, 1974). See p. 21. Strawson speaks in terms of “specification” rather than “reference,” but that is his terminology for the relation between singular terms and their bearers (“particulars”) as well.

*reference<sub>K</sub>*, will take as its range a distinctive ontological kind *K* and will comply with the principle that no two expressions of distinct syntactic category can *refer<sub>K</sub>* to an entity of that kind. Accordingly, no two expressions of distinct syntactic category will *refer<sub>OBJECT</sub>* to the same object; no two expressions of distinct syntactic category will *refer<sub>CONCEPT</sub>* to the same concept; no two expressions of distinct syntactic category will *refer<sub>FUNCTION</sub>* to the same function, and so on.

It is easy to see that none of these principles, individually or collectively, gives rise to a concept *horse*-type paradox. The reason is that while they require that expressions of distinct syntactic category cannot co-refer under any particular reference relation, *reference<sub>K</sub>*, they say nothing to prohibit their co-reference under *distinct* reference relations, *reference<sub>K</sub>* and *reference<sub>J</sub>*. So part of the general idea underlying the original Reference Principle may still consistently be affirmed. It is still perfectly fine to say that, for any particular reference relation *reference<sub>K</sub>*, two expressions are apt to co-refer<sub>K</sub> only if they are of the same syntactic kind. But it needs to be recognized that the specific relation of co-reference for which they are apt will vary as a function of the syntactic kind concerned. And the possibility is thereby opened that since “the concept *horse*” and “...is a horse” can stand in different reference relations to the same thing, the use of the former can serve to convey the semantic value of the latter.

Our remaining work is now cut out for us. First we must ask: what if any good philosophical motive did Frege have—or is there anyway—to bring semantic theory under, in effect, an inconsistent pair of constraints? And second, since if there is a good motive, that cannot be the best way to respond to it (since at least one of the ingredient principles in the Reference Principle has to be dropped), we must ask: which one, and why?

#### V. THE UNITY OF THE PROPOSITION

There is little room for doubt, so we have argued, that Frege endorsed the Reference Principle. Our first question is whether he had—or whether there is anyway—any compelling reason to do so. The question splits, in effect, into two. On the one hand, it may seem that Frege’s commitment to the Principle is integral to his whole approach to ontological questions—to his belief that distinctions among kinds of entity must be understood in terms of a prior division of expressions into different logical types. But on the other hand, some of his interpreters have claimed that Frege’s adoption of the Reference Principle is prerequisite for the explanation that he wants to give—perhaps, indeed, for any good explanation—of the “unity of the proposition.” We shall take up the first question—in effect, whether endorsement

of the Reference Principle is forced by what is often called the Syntactic Priority thesis—in our next section. But first, what of the “unity of the proposition”?

One commentator who holds that the quest for a satisfactory account of the unity of the proposition not only did move Frege, but did so quite properly, is Harold Noonan. After acknowledging that, for the greater part of “On Concept and Object,”

...Frege’s insistence on accepting whatever consequences of the absolute divide between concept and object may be deduced...seems more like a stubborn dogmatism than anything else,

Noonan avers that

...at last, at the end of the article, Frege reveals what lies behind his insistence...

Namely,

...the traditional [problem] of the unity of the proposition: what distinguishes a proposition, a sentence which expresses a thought, from a mere list of names? Frege’s solution is that a sentence, unlike a list of names, exhibits patterns, which themselves have to be understood as having semantic values. Thus the sentence ‘the number 2 is prime’ exhibits a pattern ‘x is prime’, the pattern exhibited by any sentence consisting of a proper name followed by the words ‘is prime’. And it is this pattern which is the name of the concept *prime*, for it is only by recognizing a sentence as exhibiting it that we can recognize it as a predication of the concept *prime* of an object. But, now, we cannot replace this pattern by the expression ‘the concept *prime*’ or any quotable part of a sentence. It simply makes no sense to speak of replacing a *pattern* exhibited by a sentence by a quotable *part* of a sentence. (We can replace the pattern by another, that is, rearrange the parts, or preserving the pattern, replace some or all of the parts by other parts, but that is all). *A fortiori* we cannot make such a replacement *salva veritate*. But it has no significance to speak of two expressions as having the same reference unless one can be substituted for the other *salva veritate*. Thus we cannot think of ‘the concept *prime*’, or any other quotable expression, as another (saturated) name of what is also named by the predicate ‘x is prime’.<sup>39</sup>

Let us set aside the question whether Noonan could formulate this suggestion without, as he actually does, violating the very proscription against singular reference to what a predicate stands for which he rightly takes to be central to Frege’s doctrine. The central thought is that the

<sup>39</sup>Noonan, *op. cit.*, pp. 162–63.

impossibility of referring to a concept by means of a proper name such as “the concept *prime*” or “what ‘is prime’ stands for,” and so on, is integral to Frege’s solution to the problem of the unity of the proposition.<sup>40</sup>

Well, what exactly *is* that problem, and how exactly is the Reference Principle integral to anything that might plausibly be taken to be Frege’s solution to it? Noonan’s account seems to be this: if we are to explain how, for example, a sentence such as “That man is running” avoids degeneration into a mere list of individually meaningful expressions, we must recognize that it configures expressions of two fundamentally different types—proper names (or singular terms), which are complete expressions, and predicates or relational expressions, which are incomplete. Indeed, his thought is, even to speak here of two types of *expression* is strictly incorrect and potentially quite misleading—since the crucial point is that a well-formed sentence exhibits a *pattern*, and while this pattern involves expressions, it is not itself a further, quotable expression. In contrast with a mere list, which comprises just singular terms for the items listed, we must reckon among the components of a meaningful sentence at least one incomplete expression-pattern, referring to a concept (property) or relation, such as “...is running” or “...is greater than \_\_\_\_.” But then, given that two sentence-components cannot share their reference unless they are interchangeable *salva veritate* (so, *a fortiori*, interchangeable *salva congruitate*), it follows that that concept or relation cannot be the referent of any singular term, such as “running” or “the property of running” or “what ‘is running’ stands for.”

The first thing to remark about this is that, as an argument for the indispensability of the Reference Principle, it simply begs the question. For the outlined “solution,” such as it is, to the unity problem is accomplished purely by the invocation of the distinction between complete and incomplete expressions, or “patterns”: a subject-predicate sentence, for example, differs from a list in that the complete expression it contains fills out, or saturates, the pattern represented by the predicate. When Noonan goes on to write

It simply makes no sense to speak of replacing a *pattern* exhibited by a sentence by a quotable *part* of a sentence....*A fortiori* we cannot make

<sup>40</sup> Dummett makes a similar claim:

A concept and an object, or a relation and two objects, need no glue to fit them together: they fit together naturally, in a way we can think of as analogous to that in which a predicate and a proper name, or a relational expression and two proper names, fit together to form a sentence. And this will seem to us natural and unproblematic as soon as we grasp that we can think of a concept *only* as the referent of a predicate, and of a relation *only* as the referent of a relational expression. (Dummett, *op. cit.*, pp. 174–75)

such a replacement *salva veritate*. But it has no significance to speak of two expressions as having the same reference unless one can be substituted for the other *salva veritate*. Thus we cannot think of 'the concept *prime*', or any other quotable expression, as another (saturated) name of what is also named by the predicate 'x is prime',

he merely draws out a consequence of this solution that makes *independent* appeal to the Reference Principle, rather than in any way supporting it or showing that it is an indispensable part of the solution. No motivation for the Principle itself is provided.

But in any case, what exactly is "the traditional problem of the unity of the proposition"? Noonan seems to take it that the problem Frege wanted to address is that of explaining how a *sentence* differs from a mere list. But can that be a full account of the matter? For Frege, a proposition, or thought, is rather the *sense* of a declarative sentence, and as such quite different from the sentence that expresses it. Noonan's question is no doubt a perfectly good question. But a straightforward and perfectly good answer to it is precisely that a sentence expresses a proposition, or thought (that is, says something truth-apt), whereas a list does not.<sup>41</sup>

So did Noonan ask the right question? There are passages in Frege which suggest that he discerns a *deeper* question about propositions, or thoughts, and about how to explain *their* unity, as distinct from that of sentences which express them.<sup>42</sup> Might it be that it is the treatment of this deeper question that somehow imposes the Reference Principle?

<sup>41</sup> Of course, someone may, in context, say something by uttering a string of words forming a list, such as "Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune," or even a single word, such as "Elba"—the context being supplied by some prior question ("What are the planets in our solar system?" "On which Mediterranean island was Napoleon exiled?"). More interestingly, the context may be nonlinguistic—we are out in the bush, hoping to bag materials for a rabbit stew, and our guide points to some small paw marks in the muddy footpath and utters "gavagai." We understand him to be saying something along the lines of "Rabbits were here recently!" Talk of a one-word sentence in such cases is potentially misleading—what expresses our guide's proposition is not the single word "gavagai," but its utterance *in that mutually understood context*. The context, we might say, is part of the means whereby the proposition is expressed. Something like this happens regularly with indexical expressions, as Frege observed, in his late essay on thoughts: "...if he now says 'I was wounded', he must use 'I' in a sense which can be grasped by others, perhaps in the sense of 'he who is speaking to you at this moment'; by doing this he makes the conditions accompanying his utterance serve towards the expression of a thought." Frege, "Thoughts," *Beiträge zur Philosophie des deutschen Idealismus*, 1 (1918–19): 58–77; translated by Geach and R. H. Stoothoff in Frege, *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness (New York: Blackwell, 1984), pp. 351–72, see pp. 359–60.

<sup>42</sup> See for example Frege, "On Concept and Object," p. 54: "...not all the parts of a thought can be complete; at least one must be unsaturated, or predicative, otherwise they could not hold together."

This is very foggy territory. But there *is* an immediate further question we can press about *sentences*, as opposed to lists. We can ask: what is it about a sentence that *enables* it to express a proposition—that is, to say something? A commonsensical answer is that a sentence, in contrast with a list, will normally be composed of two or more expressions of *different*, complementary semantic kinds—in the simplest case, of names or singular terms and predicates or (finite parts of) verbs.<sup>43</sup> We will have, in that case, at least one expression which serves to indicate *what* we are saying something *about*, and another which serves to express *what we are saying about* the thing(s) we are talking about.

Now, if we press this commonsensical answer, we do uncover a natural-seeming motivation for something approaching Frege's overall view. For the need for at least two semantically complementary kinds of expression allows of a straightforward extrapolation to nonlinguistic entities: if there is to be any such a thing as a singular proposition—something that essentially can be stated, or thought—as opposed to nonpropositional things that are merely mentionable, or listable, we need to recognize, or “introduce,”<sup>44</sup> at least two kinds of item. The proposition exists only if it can be stated, or thought; and for there to be such a stating, or thinking, there needs to be both something *of which* something can be said, or thought, and something apt to be *said of*, or *thought about* it. The complementary character of the respective semantic roles of singular term and predicate which allows them to combine in the expression of a singular proposition may thus quite naturally be taken to call for a corresponding distinction in the kinds of thing that provide their respective semantic values, without which there is no proposition to express.

This simple train of thought impresses as perfectly in keeping with Frege's overall way of thinking about his ontological categories. Logico-semantic distinctions at the level of language are taken to be the model and explanatory ground of distinctions at the level

<sup>43</sup> See Aristotle, *De Interpretatione*, 16<sup>a</sup>12–18: “...falsity and truth have to do with combination and separation. Thus names and verbs by themselves—for instance ‘man’ or ‘white’ when nothing further is added—are like the thoughts that are without combination and separation; for so far they are neither true nor false. A sign of this is that even ‘goat-stag’ signifies something but not, as yet, anything true or false—unless ‘is’ or ‘is not’ is added (either simply or with reference to time).” The translation is Ackrill's in *Categories and De Interpretatione*, translated with notes by J. L. Ackrill (Oxford, UK: Clarendon, 1963). Talk of “thoughts that are without combination or separation” is perhaps somewhat unfortunate, since thoughts precisely are things apt to be true or false—“thought-elements” would have been better.

<sup>44</sup> Here, we intend “introduce” in something close to the sense in which Strawson uses the word in his *Individuals: An Essay in Descriptive Metaphysics* (London, UK: Methuen, 1959), chapter 5. See, for example, page 146 and following.

of *Bedeutung*. But does it lead to the Reference Principle? To argue, in the way adumbrated, that we need a distinction at the level of *Bedeutung* to reflect the distinction between proper name and predicate at the level of logical syntax is one thing; to claim that the *Bedeutungen* of predicates are, just by virtue of being that, ineligible to be referred to by any other kind of expression, is something else. It is the latter that needs to be supported if there is to be an argument in the vicinity for the Reference Principle. No support for it is evident in the natural train of thought just outlined.

But perhaps it will be felt that we are still putting off the *deep* question about unity. That was: how do the components of *the thought itself* hang together? A thought is essentially composite; yet its components come fused into a thinkable content, rather than as a potpourri of discrete ideas. What bonds them? What unifies them into a proposition?

Well, we confess to some diffidence whether this is a question in good standing.<sup>45</sup> But suppose it is. Even so, there is no clear prospective track to the conclusion that the Reference Principle might be an indispensable part of any satisfactory response to it. For Frege, the unity of the proposition is a unity at the level of *sense*. The components of a thought are senses. If the question is how the components of a thought somehow hang together, we need to say something about the combinatorial potentialities of different kinds of sense. It is plausible enough that that would involve drawing a distinction between the kinds of sense respectively possessed by proper names on the one hand and by predicates and relational expressions on the other. But the sticking point is still going to be the question why the *Bedeutung* of a predicate, a concept or property, cannot be thought about in two quite different ways, one corresponding to its role as a predicable—the way of thinking about it that is apt to figure in a singular thought of some object that it falls under the concept, or has the property, in question, and the other corresponding to its role as the object of thought to which is ascribed some higher-order property. If this duality of ways of thinking of a concept,

<sup>45</sup> One reason for diffidence is the want of any clear and compelling formulation of a problem about unity going beyond the issues on which we have touched here. A quite independent reason for doubting that, in spite of occasional passages which encourage the contrary view (such as that quoted in note 40), Frege is properly interpreted as confronting any deeper problem of unity is that he is best understood as adopting a “top-down” approach on which complete thoughts are basic—the sense and reference of sub-sentential expressions being explained in terms of their contribution to those of complete sentences. See Mark Textor, “Unsaturatedness: Wittgenstein’s Challenge, Frege’s Answer,” *Proceedings of the Aristotelian Society*, n. s., cix (2009): 61–82, see pp. 62–63; and Textor, *The Routledge Guidebook on Frege on Sense and Reference* (New York: Routledge, 2010), pp. 68–74 and 93–102.

or property, is possible, then there should be a corresponding duality of ways of speaking of it. In that case, the Reference Principle will be an unmotivated proscription and has no bearing on the unity of the proposition. What is the argument that such a duality is not possible?

As we said, this is foggy territory. Our conclusion can only be provisional: that for all we can make of “the traditional problem of the unity of the proposition,” there is no reason to think that a satisfactory response will call for adoption of the Reference Principle. Maybe it is tidy—simplest—to think of the complete/incomplete distinction as applying isomorphically at each of the three levels: expressions, sense, reference. That is what Frege *did* think. If we opt for this isomorphism, we can say that a sentence is distinguished from a list by being an appropriate concatenation of complete and incomplete expressions; and that a thought is distinguished from a sequence of ideas by being composed of a suitable mixture of complete and incomplete senses. But it is only if extending the distinction between completeness and incompleteness down to the level of *Bedeutung*, and there aligning it with those operating at the level of expressions and senses respectively, is not (merely) natural but *required*, that the Reference Principle will issue as a consequence. And how can that be required, when the upshot of doing so is paradox?

#### VI. SYNTACTIC PRIORITY WITHOUT THE REFERENCE PRINCIPLE

Let us take stock. At the heart of Frege’s approach to ontology lies his belief—we would like to say: insight—that ontological categories are to be explained in terms of a prior division of logical types of expressions. What it is to be an entity of a certain kind—at the most fundamental level, an object or a function—is to be explained in terms of the type of expression that is apt to refer to entities of that kind. Now, undeniably, the simplest and seemingly most straightforward implementation of this idea has it that objects are what singular terms stand for, and properties and relations, and more generally functions, are what predicates, and more generally functional expressions, stand for. Ontological categories are paired off with types of expression in such a way that entities in any given category are potentially referents of one and only one syntactic type of expression, where syntactic type is determined by congruity profile—that is, expressions belong to the same syntactic type if and only if they are interchangeable *salva congruitate*. In sections I and II, we rehearsed a very strong, if not quite conclusive, case that Frege himself endorsed this implementation of his basic idea. Thus a commitment to the Reference Principle appears to be built into, and perhaps enjoined by, Frege’s approach to ontology via the logical analysis of language.

However, as we have also seen, the resultant position is radically unstable. The very principle which appears integral to Frege's ontology leads, in conjunction with other premises which Frege can hardly deny, straight into the paradox of the concept *horse*. There is—or so we argued in section III—no satisfactory way out of, or around, that paradox which does not involve rejecting some part of Frege's theory. To make a bad situation even worse (if that were possible), the Reference Principle entails, as we argued in section IV, that there can be no single type-neutral semantic relation of reference—so the very statement of Frege's theory suggested above is anyway incoherent.

Clearly, and especially in view of this last point, the Reference Principle stands out as a prime candidate for rejection or revision. But how much, if any, of Frege's theory can survive its rejection? How, if at all, can the main point of Frege's approach to ontology—the priority of logico-syntactic over ontological categories—be rescued from the wreckage? The question is not, of course, of merely historical interest. The Syntactic Priority thesis is a foundational part of the deflationary, or “minimalist” form of platonism that features in contemporary debate in the philosophy of mathematics as a very important, if controversial, response to the so-called Benacerraf Dilemma—the problem of squaring a plausible semantics for mathematical languages with a plausible, naturalistic mathematical epistemology.<sup>46</sup> Is there some refined version of Frege's general approach to ontological questions which salvages a significant portion of his view and conserves its philosophical advantages while avoiding paradox? In the remainder of the paper, we shall consider two possible attempts at an affirmative answer.

As we saw in section IV, the Reference Principle is in effect the conjunction of two quite distinct—and indeed jointly inconsistent—principles, namely:

*Single Relation:* Some *one reference relation* uniformly connects expressions of each syntactic type with the kinds of entity that provide their respective semantic values.

*Type-Kind Uniqueness:* *Syntactic types* of expression correlate one-to-one with the *kinds of entity* among which their tokens are eligible to refer.

If what we have argued thus far is right, any solution to our problem must involve rejecting, or at least modifying, one or other of these

<sup>46</sup> For the dilemma, see Paul Benacerraf, “Mathematical Truth,” this JOURNAL, LXX, 19 (Nov. 8, 1973): 661–79; and for a concise statement of the deflationary platonist response, Hale and Wright, “Benacerraf's Dilemma Revisited,” *European Journal of Philosophy*, x, 1 (April 2002): 101–29, especially §§5,6.

two principles, and the question now confronting us may therefore seem simple enough: which one? We will suggest that the answer may not be quite as simple. Still, of the two approaches to be considered, one centrally involves rejecting the *Single Relation* principle, while the other clearly requires modifying *Type-Kind Uniqueness*. In brief outline, these are as follows.

*Rejecting the Single Relation Principle.* In section IV we rehearsed an argument to show that, if the *Type-Kind Uniqueness* principle is retained, the *Single Relation* principle must go. Essentially the same argument was first presented in Wright 1998.<sup>47</sup> The negative moral there drawn was that Frege was mistaken in his assumption that reference (*Bedeutung*) can play a key role in the semantics of predication, just as it does in the semantics of singular terms (Frege's *Eigennamen*)—that just as singular terms have objects as their referents, so predicates have concepts as theirs.<sup>48</sup> The positive proposal may be summarized, in bare essentials, as follows.

We should distinguish, just as is done in ordinary intuitive thought, between two contrasted elements involved in the simplest acts of saying something—*naming*, which one does by means of a singular term, and *describing*, which one does by means of a predicate. If we take the relation that holds between a singular term and its semantic value to be *reference*, we should recognize a different relation as holding between a predicate and the nonlinguistic entity that is its semantic value in ordinary acts of predication—that is, a Fregean concept, or property: a predicate does not refer to, but instead—we can say—*ascribes* a concept or property.<sup>49</sup>

This proposal is clearly consistent with retention of the Reference Principle, *as construed* in Wright's 1998 paper, which requires only that sameness of reference ensures sameness of semantic role, so that co-referential expressions are always interchangeable *salva congruitate*.<sup>50</sup> Further, nothing in the proposal precludes acknowledging that one can refer to Fregean concepts, or properties, by means of singular terms, such as "the concept *horse*" or "the property of *being wise*." The failure of predicates such as "ξ is wise" to be interchangeable with the corresponding names just means that they do not *refer* to the properties

<sup>47</sup> There are some slight differences, both of formulation and in orientation. In particular, Wright's 1998 paper effectively identifies the Reference Principle with what we are calling the Type-Kind Uniqueness principle, and takes its preservation, so understood, as one of the constraints upon a solution to Frege's problem.

<sup>48</sup> See Wright's 1998 paper, p. 84.

<sup>49</sup> *Ibid.*, pp. 86–87.

<sup>50</sup> As Wright further argues—in his 1998 paper, see pp. 87–88—it also meets the other constraints there taken to be necessary, but there is no need for details here.

which they serve to ascribe—not that those properties cannot themselves be objects of reference. And the paradox is dissolved. So far from being contradictory, such statements as that the concept *horse* is an object, that the *ascriptum* of “ $\xi$  is wise” (that is, what “ $\xi$  is wise” ascribes) is an object, and their like are straightforwardly and unproblematically *true*. For objects are, just as Fregean orthodoxy says, simply the referents of (actual or possible) names. This is perfectly consistent with those objects being also what the corresponding predicates ascribe (but do not refer to).

*Rejecting Type-Kind Uniqueness.* As we saw, there is an ostensibly quite different way to solve Frege’s problem—to reject, or modify, the Type-Kind Uniqueness principle. Of course, *simply* rejecting that principle outright is tantamount to scrapping Frege’s approach to ontology altogether. For without endorsing something like the principle, we cannot explain ontological categories in terms of a prior division of expressions into logico-syntactic types. We can no longer characterize objects as what singular terms stand for, properties and relations as what predicates stand for, and so on, and, in the absence of the Type-Kind Uniqueness principle, or something close to it, there is apparent no other way to explain what objects, properties, and so on, are in line with the basic Fregean thought that logical syntax is prior to ontology.

However, if we accept, contrary to the Type-Kind Uniqueness principle, that entities may be *referred to* by expressions belonging to different logical types, there is—or so, in a recent paper, Hale contends<sup>51</sup>—a simple and plausible modification which avoids our problem whilst preserving the essential ideas of the Fregean approach. Where there are expressions of different logical types having reference to entities of a single given kind, we distinguish between *primary* and *secondary*, or derivative, modes of reference to them. For example, while we *can* refer to properties by means of (complex) singular terms (such as “the property of being wise”), the *basic* mode of reference to them is by means of appropriately incomplete expressions (such as the predicate “...is wise”). Similarly, we *can* refer to the truth-function of conjunction, say, in just that way, using a name for it, but the basic mode of reference to it is by means of the incomplete expression: “...and —”; and so on for other cases. We may then replace the simple Fregean characterization of objects:

*Objects:* Objects are those things to which singular terms refer,

<sup>51</sup> What follows summarizes Hale, “The Bearable Lightness of Being,” *Axiomathes*, xx, 4 (2010): 399–422, see pp. 415–16. We refer to this article as “Hale 2010.”

by the slightly more complicated:

*Objects\**: Objects are those things to which singular terms *primarily* refer, making similar adjustments in the characterization of entities of other kinds. Thus in particular, we have

*Properties*<sup>n</sup>: *n*<sup>th</sup>-level properties are those things to which predicates of level *n* primarily refer.

Our revised explanations allow that entities of a given category may be referred to by expressions other than those of the type in terms of which the category is defined. Thus a property such as that of being a horse may be referred to by a singular term (as we just did), but because that is not the primary mode of reference to properties, it does not make objects of them, and so precipitates no conflict with our explanation of properties. More generally, entities of every type acknowledged in Frege's ontology may be referred to by a name or singular term; but it is only in the case of objects that that is the primary mode of reference to them. The paradox of the concept *horse* is thus avoided without prejudice to Frege's insistence that the distinction between first- and second-level functions, and so that between their respective arguments—objects and concepts (or other functions)—is absolute and “founded deep in the nature of things.”

There are, then, at least two potential lines of solution to Frege's problem, each of which promises to conserve arguably central features of his overall position. Does one, or the other, allow of an ultimately satisfactory development? We do not know the answer to that question. While each faces a range of problems, we know of no *clearly* decisive objection to either proposal. In the remainder of the paper we offer an inevitably brief and partial outline of some of the relevant issues.

#### VII. PROSPECTS (1): REJECTING THE SINGLE RELATION PRINCIPLE AND TREATING FREGEAN *INCOMPLETA* AS OBJECTS

Harold Noonan claims that there is a fatal incoherence in Wright 1998's claim that a concept can be both the *ascriptum* of a predicate and the referent of a singular term. In detail, his objection runs as follows:

...the expression ‘ascription’ was coined [by Wright] to indicate just that unequal-level relation between a predicate and a concept which is analogous semantically to the first-level relation between singular term and referent. It is, in fact, that relation expressed by: “...” applies to something if and only if it ...’ (e.g. ‘is a horse’ applies to something iff it is a horse).<sup>52</sup>

<sup>52</sup> Noonan, *op. cit.*, p. 168.

But if this is how “ascribes” is to be understood, Noonan argues,<sup>53</sup> then Wright’s position is incoherent. For he must hold that there is something which “is a horse” ascribes and to which “the concept *horse*” refers. That is, there is some *X* such that “is a horse” ascribes *X* and “the concept *horse*” refers to *X*. But “‘is a horse’ ascribes *X*” expands as “‘is a horse’ applies to something iff it *X*,” so Wright needs to hold that some instance of the schema:

“is a horse” applies to something iff it *X* and “the concept *horse*” refers to *X*

is true. But there can be no true instance, because what must replace the first occurrence of “*X*” is a first-level predicate, whereas what must replace the second occurrence must be a singular term.

But the objection is mistaken. The explanation of ascription on which it relies is Noonan’s own, *not* Wright’s. Wright’s explanation runs as follows:

For a predicate to stand in the relation of ascription to a property or concept is just this: for its sense so to relate it to that property/concept that it may be used in concatenation with an appropriate singular term to say of the bearer of that term that it has the property, or falls under the concept in question.<sup>54</sup>

Wright does not explain “ascribes” as an *unequal-level* relational expression. Noonan just *assumes* that it *must* be unequal-levelled—specifically, that it involves one argument place to be filled by a singular term and another to be filled by a first-level predicate—because it is intended to be a relation between objects (expressions) and properties, and he (following Frege) takes it that properties can *only* be designated by predicates. But he is, precisely, not entitled to do that. Wright gives no such explanation, and is not committed to the ill-fated suggestion that the relation of ascription can be expressed by an unequal-level predicate. Noonan is just not taking seriously Wright’s view that concepts/properties are a kind of object. Given that they *are* a kind of object, there is no reason why the relation of ascription should not be a relation expressed by a *first-level* predicate, expressing an object-object relation (but holding only when the second term is a special kind of object, namely, a property). Thus Wright *can* say that there is something which “is a horse” ascribes and “the concept *horse*” stands for. This has the form:

$\exists x(\text{“is a horse” ascribes } x \wedge \text{“the concept } \textit{horse}\text{” stands for } x)$

<sup>53</sup> Here we paraphrase the two paragraphs immediately following the passage just quoted.

<sup>54</sup> Wright 1998, p. 88.

For this to be true, there has indeed to be a true substitution instance—but there is, namely:

“is a horse” ascribes the property of being a horse  $\wedge$  “the concept *horse*” stands for the property of being a horse.

What is of course correct, but an entirely separate point, is that it is a cost of Wright’s solution that it takes properties to be a kind of object (and, correspondingly, it is an advantage of Hale’s that it avoids doing so). For objects are still characterized as what can be referred to by singular terms, and, Wright is affirming, properties can indeed be so referred to. And it is clear that this point generalizes—entities of every kind that Frege recognizes, and so functions of every type and level, must all be kinds of object, on Wright’s view. For all such entities can be referred to by means of suitable complex singular terms, as well as bearing some further semantic relation to expressions of the kind which Frege took to refer to them. This is utterly at odds with Frege’s insistence that the difference between complete and incomplete entities “lies deep in the nature of things,” and with the view which naturally goes with it, that *object* and *function* are distinct ontological categories.<sup>55</sup> To this—significant—extent, Wright’s 1998 proposal is un-Fregean.

Is it worse than that? As Charles Parsons has noted,<sup>56</sup> treating concepts, say, as a kind of object would seem to involve treating falling under a concept as a kind of *relation*: a relation, instantiation, say, that obtains between the relevant pair of objects just when one *is*, predicatively, the other. That suggests that the following principle of comprehension should hold: that where *F* is any predicate, “ $\eta$ ” expresses the instantiation relation, and “*x*” and “*y*” range over objects,

$$(Com.) (\exists y)(\forall x) (x \eta y \leftrightarrow Fx)^{57}$$

—when and only when an object is *F*, it will stand in the instantiation relation to the object that is the concept *F*. But *Com.* is just the familiarly inconsistent universal comprehension schema. To disclose its inconsistency, we have only to take *F* as “does not instantiate itself.”

This is a vivid reminder that any satisfactory treatment of the semantics of predication must sooner or later reckon with the semantic paradoxes. But it is fair to reply that the problem is hardly special to treatments that construe concepts, or properties, as a kind of object. It arises, as Parsons in effect observes, for any proposal that views

<sup>55</sup> At least if we assume that different categories must be disjoint.

<sup>56</sup> See §5, “Is whatever is an object?” of Charles Parsons, *Mathematical Thought and Its Objects* (New York: Cambridge, 2008), pp. 13–22.

<sup>57</sup> *Ibid.*, p. 17.

concepts, or properties, as *associated* one-to-one with representative objects—sets, or extensions, for example—as Frege’s own view notoriously did. On any such view, it is equally plausible to say that for  $x$  to be  $F$  is for it to stand in a certain relation—membership, perhaps—to an object that distinctively so corresponds to  $F$ . So *Com.* is again apparently well motivated on any such view. Thus it appears that nothing less than a thoroughgoing repudiation of any such general association between concepts and objects is going to avoid the difficulty. One radical such repudiation would involve denying that concepts either are, or are distinctively associated with, objects at all—in effect, to repudiate the notion of the extension of a concept altogether. This was perhaps the view to which Frege himself eventually came, late in his career. Otherwise, the project must be to impose some well-motivated form of restriction on *Com.* All the well-tryed restrictions available to theories of sets that wish to retain some form of close tie between set-membership and predication will be available to a proponent of Wright 1998. In general, there is no reason to expect that a solution to the paradox of “the concept *horse*” should bring a solution to the semantic paradoxes with it, and it is thus no weakness in a proposal to the former purpose that it fails to address the latter—(though it would undeniably be a considerable strength if a proposal could somehow address both issues in an integrated way).

Paradox and naïve universal comprehension aside, there is a well-known general concern<sup>58</sup> about the very idea that an object’s falling under a concept is its standing in a certain relation to that concept: in effect, that a vicious regress is thereby launched, since we will then, *pari passu*, be obliged to conceive of the obtaining of that relation between the object and the concept as itself a matter of the obtaining of a three-term relation between the object, the concept, and the instantiation relation...and so on indefinitely. But, so runs the concern, to treat concepts (and relations) as a kind of object is to make the relational construal of an object’s instantiation of a concept, and the ensuing regress, inevitable.

This is a version of Bradley’s Regress, and it would take another full-length discussion to explore the issues it raises. Here it must suffice to note two points. First, it is by no means evident that the regress is vicious. We owe to Frege the insight that one and the same thought may allow of decomposition into distinct logical forms: the thought that Socrates is wise, for instance, may be regarded both as a first-level predication of wisdom of Socrates, and as a second-level

<sup>58</sup> Also noted in *ibid.* at p. 13.

predication, of “Socratising,” or applying to Socrates, of wisdom. It is open to us to take a similar view of the successive contents in the regress: *a*’s being *F* does indeed consist in *a*’s standing in a certain relation—instantiation—to the concept *F*, and this in turn does indeed consist/involve in *a*, the concept *F*, and the instantiation relation themselves standing in a certain ternary relation, namely, that of *a* and *F* being related by the instantiation relation, and so on. Why suppose there is anything wrong with that?

Second, if there is indeed a problem, it is not clear either that it is forced on us specifically by treating concepts as objects or that it is really resolved by Frege’s proposal not to do so. Let it be that concepts are not objects. How exactly does that obviate any construal of *a*’s being *F* as involving a relation between *a* and the concept *F*? The putative problem is generated by the insinuation of a relation; but not all relations are restricted to objects. There are unequal-levelled relations in any case, so the problem, if it is there at all, does not go away merely at the insistence that concepts are not to be found among the objects. Nor does it go away with the rhetoric of “incompleteness,” “unsaturatedness,” as so on. An object’s saturation of an incomplete entity looks, *prima facie*, every bit as much a relational matter as a jigsaw piece’s fitting into a vacant space in a partially completed puzzle. In effect, shorn of the metaphors, Frege’s “solution” to the problem of the regress amounts to no more than an insistence that we must not conceive of instantiation relationally. In the previous paragraph, we raised a doubt whether others of his own ideas do not in effect show that there is no real need for that insistence. But if one is unpersuaded by that, well, it is equally open to us to insist that where an object is actually a concept, a second object’s instantiation of that concept is, likewise, not to be conceived as a relation between the objects concerned.

We do not of course for one moment suggest that these sketchy remarks constitute a treatment of the topic; only that the orthodoxy that to treat the referents of predicates and functors as objects is *eo ipso* to bring down paradox and vicious regress upon one’s head stands in need of a much better elaboration than any which, to our knowledge, it has ever received before it should seem persuasive.

A fourth concern arises with the proliferation of semantic relations that Wright’s 1998 proposal entrains. The proposal has to be implemented in a fully general fashion: it must be applied to incomplete expressions across the board—not just to predicates (of each level), but also to relational expressions (again, at every level), and to other kinds of functional expression, including term-forming functors and sentential operators. In his 1998 paper, Wright allows we can refer to

a certain function by means of “the function which takes each number to its square,” and to another by means of “the function of propositions which takes the value *True* if and only if both its arguments are true,” and that these functions, along with all other functions, are objects. But we are to deny that the incomplete expressions, “the square of...” and “...and —” refer to these functions. So what do they do to them? “The square of 17” does not *ascribe* being a square to anything, and “grass is green and the sky is blue” does not ascribe being a conjunction to anything. It seems that we must find, or postulate, further relations between functional expressions and functions, analogous to but distinct from ascription. Since Frege’s ontology includes a potential infinity of kinds of function, it seems that Wright’s proposal will require a matching infinity of distinct semantic relations, each replacing the original single Fregean reference relation in application to a relevant class of expression-function pairings.

That is not, so far, an obviously intolerable consequence, but it leads to a significant epistemological concern. How are these relations to be *grasped*? The aim is to preserve the order of explanation from logico-syntactic types to ontological kinds. But we can no longer characterize each kind as that kind of thing for which the expressions of such-and-such a type are distinctively apt to refer. Rather, concepts, or properties, are that kind of thing which predicates are distinctively apt to *ascribe*; and in general, things of kind K are that kind of thing for which expressions of such-and-such a type are distinctively apt to...well, apt to stand in such-and-such a K-specific words-world relation to. And now it appears that, in order for these characterizations to work, the called-for multifarious kinds of relation must be explained in advance and independently; for even if a relevant expression-type, E, is individuated in advance, we can hardly understand a characterization of a putative ontological kind as: that kind of thing to which expressions of type E stand in relation P if we do not know which relation P is. Frege’s characterization of, for example, concepts as the referents of predicates is intelligible just insofar as we know what predicates are and have an understanding of the uniform relation of reference on which it calls. But Wright 1998 forfeits any such uniform conception. Now each characterization of an ontological kind appeals to a special semantic relation peculiar to that kind, and can succeed as an explanation only if we already have some adequate conception of that relation. Indeed, it appears that Wright 1998 actually offers no model of how revised but still broadly Fregean explanations of the ontological categories are to run.

This is a serious objection. But perhaps it allows of a response. If we reject the Single Relation principle, we must deny that, where

$E^1$  and  $E^2$  are distinct types of expression, and  $K^1$  and  $K^2$  distinct kinds of nonlinguistic entity, there is any relation  $R$  which holds between expressions of type  $E^1$  and entities of kind  $K^1$  and between expressions of type  $E^2$  and entities of kind  $K^2$ . However, every expression, whatever its type, has—barring accidents, as it were<sup>59</sup>—some entity of some kind as its semantic value. Thus singular terms have objects as their semantic values, first-level predicates have properties of objects as their semantic values, and more generally, functional expressions of each given type and level have suitable functions as their semantic values. So there is, after all, a universal semantic relation, whose domain comprises expressions of all types and whose range includes entities of every kind—namely, the relation expressed by “... has — as its semantic value.”

A proponent of Wright 1998 has no good motive to deny that there is any such universal semantic relation. For since entities of every kind are now one and all available to be referred to by singular terms, what fills in the gap marked by “—” can always be an appropriate singular term: we can perfectly correctly say, for example, both that “the concept *horse*” has the concept *horse* as its semantic value and that “...is a horse” has the concept *horse* as its semantic value. What the point brings out is that the Single Relation principle that *is* being denied needs a more judicious formulation. What it essentially involves—and what should be being denied by Wright 1998—is that there is some one semantic relation,  $R$ , with the following property: that if  $E$  stands in  $R$  to  $a$ , then  $E$  is a singular term if and only if  $a$  is an object; and  $E$  is a predicate if and only if  $a$  is a concept/property; and  $E$  is a functor if and only if  $a$  is a function... In short, there is no *type-kind bijective but universal* semantic relation. But denying this is perfectly consistent with admitting the universal semantic relation adduced above.

The suggestion we want to canvass is that this universal semantic relation may be exploited to finesse the explanatory problem generated by the proliferation of kind-specific semantic relations that Wright 1998 leads to. We start with a grasp of the logico-syntactic distinction between, say, singular terms and predicates, and the general idea of an expression’s having something as a semantic value: can we move from there to a grasp both of the distinction between object and concept and an appreciation of the distinct relations—reference and ascription, respectively—in which their proper expressions stand to them?

<sup>59</sup> For example, a singular term may fail to refer to any object, perhaps because there just happens to be no object for it to refer to, or perhaps as a matter of necessity. Likewise, a predicate may fail to ascribe any property, say, because it involves an empty singular term.

We cannot explore the matter properly here. But we will outline some initial moves. The Fregean explanations:<sup>60</sup>

*Objects* are what all and only (possible) singular terms refer to; and  
*Concepts* are what all and only (possible) predicates refer to,

serve for one who has grasped the syntactic distinction between term and predicate and the (alleged) general notion of reference to accomplish two things: to fix the notions of object and concept, and to coordinate the difference in syntactic function between terms and predicates with the categorical distinction in their semantic values. It is in referring to an object that an expression is fitted to discharge the role of singular term; and it is in referring to a concept that an expression is fitted to discharge the role of a predicate. What we need from a development of Wright 1998 is correspondingly an account of what objects are, an account of what concepts are and how they are distinguished from objects in general, and something to coordinate the syntactic distinction between singular term and predicate with reference to an object and ascription of a concept, respectively. We will have accomplished it if we can set up a situation in which the following explanations are fully intelligible:

*Objects* are what all and only (possible) singular terms *refer to*; and  
*Concepts* are what all and only (possible) predicates *ascribe*.

Now, Wright 1998 denies us the general notion of reference appealed to in the Fregean explanations. All we have is the universal semantic relation introduced above. And if we plug that into the Fregean explanations, we get the falsehoods

*Objects* are what all and only (possible) singular terms *have as semantic value*; and  
*Concepts* are what all and only (possible) predicates *have as semantic value*

—falsehoods because now some singular terms have concepts as their semantic value (for example, “the concept *horse*”), and some predicates (for example, “...is a horse”) share their semantic value with a singular term. We can restore truth by paring down to

*Objects* are what all (possible) singular terms *have as semantic value*; and  
*Concepts* are what all (possible) predicates *have as semantic value*.

But these do nothing to explain how concepts are distinguished from objects in general, nor to coordinate having a concept as semantic value with the semantic role of a predicate.

<sup>60</sup> As before, we prescind from possibilities of reference failure.

Clearly there can be no progress unless we are given more to work with than just *singular term*, *predicate*, and *has as semantic value*. But arguably we do have more to work with: we have the intuitive notions, canvassed a little earlier, of talking about something—singling it out—and characterizing it. So maybe—if not begrudged *these* “grains of salt”—a proponent of Wright 1998 can make some headway by starting with the following rubric for simultaneous characterization both of a kind and of its associated kind-specific semantic relation in terms of prior syntactic distinctions and the universal semantic relation:

Any entity is of **kind K** which *is the semantic value of a* (possible) expression, E, of *type T*, and whose bearing the specific **semantic relation, R**, to E would enable E to play *the distinctive linguistic role characteristic of expressions of type T*. Only such entities are of **kind K**.

Here the emboldened expressions mark the explananda, while those italicized comprise the materials brought to bear on the explanation. As special cases we thus have,

Any entity is **an object** which *is the semantic value of a* (possible) *singular term*, E, and whose **being referred to** by E would enable the use of E to *single out and facilitate talk about the entity in question*. Only such entities are **objects**.

And

Any entity is **a concept/property** which *is the semantic value of a* (possible) *predicate*, E, and whose **being ascribed** by E would enable the use of E to *characterize an object being talked about*. Only such entities are **concepts/properties**.

Then perhaps, building on an analogous characterization of **relations** and an association of any properly explained *functor* with a given relation, R, we might propose

Any entity is **a unary objectual function** which *is the semantic value of a* (possible) *unary objectual functor*, E, and whose **being denoted** by E would enable the application of E to a *singular term*, t, to *refer to the unique object R-related to the referent of t*. Only such entities are **objectual functions**.

We anticipate the reaction that, even if the proposed rubric of explanation does indeed extend intelligibly this far, we surely lack conceptions of “distinctive linguistic roles” to pair one-one with each of the syntactic categories that Wright 1998 will recognize. No doubt. But maybe it is enough if we have such conceptions to match with certain broad determinable categories of semantic value: relation in general, objectual function, higher-order function...The matter needs detailed discussion. Here the point we have wanted to bring out is merely that

the prospects for a broadly Fregean ontology based on Wright's 1998 proposal are much more fraught than anticipated in that paper; but that they are not, perhaps, hopeless.

VIII. PROSPECTS (2): REJECTING THE TYPE-KIND UNIQUENESS PRINCIPLE BUT RESPECTING FREGE'S DISTINCTION BETWEEN COMPLETE AND INCOMPLETE ENTITIES

By rejecting *Type-Kind Uniqueness*, Hale's 2010 proposal can allow that syntactically incongruous expressions may bear a single semantic relation to entities of different types—names, predicates, relational, and functional expressions quite generally may all be said to refer (to objects, properties, relations, and functions)—and is therefore untroubled by the problems just sketched. But there are certainly further questions about the proposal which must be satisfactorily answered before one could have any confidence that it is viable. Obvious among them is one concerning its key distinction between basic and derivative modes of reference: in what sense, exactly, is reference to a property by means of a predicate *basic*, and reference to it by means of a singular term *derivative*? The distinction is crucial to the proposal's revised characterization of the Fregean ontological categories; functions, like objects, may be the referents of singular terms, but they are distinguished from objects precisely by their being those things to which *basic reference* may be made only by means of functional expressions. Some headway may be made by observing that names of properties, relations, and functions are always complex, and typically incorporate either the corresponding predicates or functional expressions themselves or their nominalizations (for example, "the property something has if and only if it is wise," "what 'is wise' stands for," "being wise"). But while this may be a useful start, we should hope for a deeper and more general explanation.

One attractive suggestion is that complex *names* for properties and functions may always be seen as implicitly definable from the corresponding predicates or other functional expressions by means of *abstraction principles*—that is, principles of the general form:

$$\forall \alpha \forall \beta (\#(\alpha) = \#(\beta) \leftrightarrow \text{Eq}(\alpha, \beta))$$

where  $\alpha, \beta$  are variables of any single syntactic type (name, predicate, and so on),  $\#$  is a functor taking arguments of that type, and  $\text{Eq}$  an expression for an equivalence relation of appropriate type and level. First-level property abstraction will assume the more specific form:

$$\forall F \forall G (\text{the property an object } x \text{ has iff } Fx = \text{the property an object } x \text{ has iff } Gx \leftrightarrow \text{Eq}(F, G))$$

or, more compactly, using the  $\lambda$ -notation for property-abstracts:

$$\forall F \forall G (\lambda x Fx = \lambda x Gx \leftrightarrow \text{Eq}(F, G))$$

where Eq stands for some suitable second-level relation on first-level properties (the semantic values of  $F$  and  $G$ ). Intuitively, this equivalence relation will supply a necessary and sufficient condition for property identity. The demand for such a relation is, in effect, an application of Quine's doctrine, summarized in his slogan "No entity without identity."

How exactly might this assist us in explaining the way in which reference to properties by means of names is derivative—is dependent upon more basic reference by means of predicates? The thought is simple enough: the dependence shows itself in the fact that the truth-conditions of identity-statements linking *names* for properties receive a canonical explanation in terms of a *higher-level* relation whose terms are specified precisely by means of the *predicates* corresponding to those names.

So, the proposal is: a range of singular terms,  $t_1, \dots, t_n, \dots$  refers *derivatively* to a range of entities,  $E_1, \dots, E_n, \dots$ , just in case the explanation of their reference proceeds systematically in terms of the use of other expressions,  $e_1, \dots, e_n, \dots$  which are *not singular terms* and which are *already given as having their reference among*  $E_1, \dots, E_n, \dots$ . Abstraction then presents itself as an obvious and natural strategy for delivering the relevant systematic explanations.

It may be helpful at this point, and may help forestall a possible misunderstanding,<sup>61</sup> to contrast Hale's proposal with Frege's own view. As is well known, Frege held that while functions (including concepts) are incomplete entities, and so not objects, there *are* objects of a certain kind corresponding to them—namely, value-ranges (in the case of concepts, extensions). In "On Concept and Object," he complains that "a kind of necessity of language" obliges him to mention an object, when he intends a concept. It may with some plausibility be supposed that the objects to which, according to Frege, language obliges us to refer, when we really wish to speak of functions (or concepts), are value-ranges (or extensions), so that whereas a predicate (for example, "is a horse") refers to a concept, the corresponding singular term (for example, "the concept *horse*") refers to its value-range (extension). There may be some temptation to interpret Hale's proposal as much closer to Frege's view than it really is. For it may be supposed that explaining the derivative or secondary character of reference to concepts by means of singular terms by way of abstraction

<sup>61</sup> We are indebted to the editors for bringing the possibility of this kind of misunderstanding to our attention.

principles, as proposed, amounts to claiming that while predicates refer to concepts *directly*, singular terms refer to them only *indirectly*, this indirect reference being *mediated* by *direct* reference to corresponding abstract objects.

If this were Hale's proposal, it might be expected to run into trouble, just as Frege's view does, when it comes to characterizing the relation between concepts and the objects corresponding to them. For Frege, the relation between functions and their value-ranges is governed by his Basic Law V, which is of course inconsistent. But this is *not* the proposal. The proposal is not that singular reference to concepts is derivative because it is indirect, being mediated by reference to objects systematically correlated with concepts—rather, it is no less direct, but nevertheless derivative because dependent upon reference to concepts as effected by predicates. This dependence is highlighted by property-abstraction principles, which serve, according to the proposal, to introduce new singular terms standing, not for objects of any kind, but for concepts—the very same concepts to which basic reference is effected by means of the predicates which may occupy the argument places on their right-hand sides. There is, therefore, no question of the need to characterize the relation between the concepts for which predicates stand and the objects to which the correlated abstract singular terms refer—for there are no such objects.

That is not to deny that the proposal faces other large, and hard, questions, including a potentially troublesome question concerning its consistency. Most obviously, perhaps, we need to ask: just how, for the target case of properties, should the abstraction's equivalence relation be specified? What is required for the identity of properties, or relations, or functions in general? There may be good reasons to depart from Frege's purely extensional conception of functions. But any plausible alternative will at least require predicates to be coextensive if they are to stand for the same property, even if that condition is not taken to be sufficient. And however strong the replacement condition,  $\text{Eq}(F,G)$ , it will surely be satisfied by any pair  $\langle F,F \rangle$ , whence the existence of  $\lambda F$  will straightforwardly follow. But this is enough to ensure that, if unrestrained, not only plain coextensiveness—that is,  $\forall F \forall G (\lambda x Fx = \lambda x Gx \leftrightarrow \forall x (Fx \leftrightarrow Gx))$ —but any plausible strengthening of it will lead to contradiction. For pending imposition of a restriction somewhere, nothing will preclude defining a Russell property  $R$  by:  $Rx =_{\text{def}} \exists F (x = \lambda F \wedge \neg Fx)$  and then a Russell object by  $r = \lambda R$ , and Russell's contradiction will be derivable. It is a nice question, too large for discussion here, what is the weakest—and therefore most preferable—restriction that will secure consistency.

Less obviously, perhaps, but no less important: what are the further implications of Hale 2010 for our understanding of higher-order abstraction in general? Higher-order abstraction principles are normally taken as defining functions from properties or Fregean concepts to *objects*. To take the most widely discussed example,

*Hume's principle*:  $\text{NxFx} = \text{NxGx} \leftrightarrow \exists R R$  one-one correlates  $F$  and  $G$ ,

is normally taken to define a function—the number function—which takes properties to cardinal numbers, conceived as a species of object. But on the proposal about derivative reference and its ontological significance presently under discussion, this interpretation comes into question. For if we are going to claim that complex property terms such as “the property of being wise,” while singular, nevertheless refer only derivatively, and that their reference is to properties rather than objects, and we justify this claim by appeal to the fact that the truth-conditions for the relevant identities are canonically given using a second-level relation on properties, given via predicates, rather than a first-level relation on objects, then why, if it would, would it be wrong to take exactly the same view of number-terms as they figure in Hume's principle?

It would be premature to draw the conclusion that, on this way of developing Hale 2010, numbers will indeed not be objects—as Frege, and we ourselves in our previous defenses of our neo-Fregean program have held—but properties instead. More thought needs to be expended on the best way of conceiving the notion of derivative singular reference, and there may be relevant distinctions to take account of among higher-order abstractions, whose effect is that only some of them serve to introduce merely derivatively referring singular terms. But the possibility of an upshot that corrects the conception of numbers, and other higher-order abstracts, as Fregean objects ought not, on reflection, be viewed as surprising. For as far as the notion of object is concerned, the distinctive twist in the proposal of Hale 2010 is exactly that being the referent of a singular term is not, after all, sufficient for objecthood—to be an object is to be something which not only can *but can only* be the referent of a singular term. And whether the cardinal numbers—the referents of terms of the type “the number of  $x$  such that  $Fx$ ” introduced via Hume's principle—can *only* be referred to by singular terms is exactly the point at issue.

#### IX. CONCLUSION

It should be apparent that much further work is needed if it is to be determined whether either, or both, of the two proposals we have explored is viable, whether either is to be preferred, and what other

possibilities there might be. But there are some lessons—some philosophical horse sense—that can be safely drawn from the discussion to this point. They are:

That the paradox of the concept *horse* flows from elements in Frege's thought that are absolutely integral to his general approach to ontology and to his philosophy of mathematics.

That the paradox needs to be resolved—there is no theoretically defensible stance of living with it.

Above all, that while it is an open question whether there is any satisfactory resolution of the paradox consistent with the aspect of Frege's approach that is the most important and of most contemporary relevance—the ranking of logico-syntactic distinctions as metaphysically and epistemologically prior to those of ontological category—the prospects for an affirmative answer, in either of two quite different ways, are not discouraging.

*Postscript.* After the present discussion had gone to press, a forthcoming paper of Fraser MacBride came to our notice ("Impure Reference: A Way around the Concept *Horse* Paradox," *Philosophical Perspectives*, xxv, 1 (December 2011): 297–312) of which we should certainly have wished to include a substantive discussion had we known of it sooner. MacBride notes that there are, plausibly, natural-language counterexamples to the Reference Principle (in the form in which it is stated in Wright 1998)—cross-substitution of "me" for "I" in "I am hungry," for example, results in a compromise of *congruitas* yet the terms involved are co-referential—and attributes this kind of failure to what he characterizes as *impure reference*: the terms "me" and "I," though alike in reference, incorporate additional syntactic information (flagged by their accusative and nominative cases, respectively) which constrains the kind of part they can respectively play in a well-formed construction. MacBride's suggestion is that something similar holds for predicates which, in addition to their role of reference to concepts—a role they do share, according to his proposal, with the corresponding singular terms that Frege found troublesome—incorporate additional syntactic information concerning how they are to be deployed in combination with other expressions to express a well-formed thought. Because of possibilities of impure reference, the Reference Principle should never, in MacBride's view, have seemed attractive without qualification. The strongest correct such principle is rather:

If  $\alpha$  and  $\beta$  are (i) co-referential expressions and (ii) *purely* referential then (iii)  $\alpha$  and  $\beta$  are substitutable in extensional contexts *salva veritate* and in all contexts *salva congruitate*.

And since predicates are impurely referential, this principle poses no barrier to the co-reference of "...is a horse" and "the concept *horse*."

Provided that the notion of impure reference can be sufficiently elucidated to sustain the weight that MacBride is placing upon it, this offers a different way out of the paradox from Wright 1998 and Hale 2010 (though it sides with Hale in rejecting Type-Kind Uniqueness while retaining Single Relation). We have no space to discuss it adequately here, but we can summarize our main reactions. One ground for concern is that the range of purely referential expressions in MacBride's sense seems likely to turn out to be so severely restricted that the principle will apply only to singular terms, and to a very narrow subclass of them at that. But if, as Frege thinks, reference is to be ascribed to expressions of all logical types, one would expect the Reference Principle to apply equally to expressions of other types—for example, co-extensive predicates. The principal drawback with MacBride's idea, however, for anyone in sympathy with Frege's approach to ontology, is that it is offered without (it seems) any thought being given to the impact involved on any proposed order of metaphysical explanation that is to run from logico-syntactic categories to ontological ones. Indeed, on MacBride's view, it appears that the distinction between singular term and predicate is merely a *grammatical* distinction, rather than one of ontological significance, on all fours with, say, the distinction between "Brutus" and "Brutum" in Latin. MacBride offers a viable way of finessing the paradox of "the concept *horse*" only for someone who has no vestige of sympathy for the considerations that generated the problem for Frege in the first place.

BOB HALE

The University of Sheffield

CRISPIN WRIGHT

New York University and the  
Northern Institute of Philosophy,  
University of Aberdeen