For R. Dietz and S. Morruzzi, eds., *Cuts and Clouds*, Oxford University Press forthcoming

The Illusion of Higher-Order Vagueness

CRISPIN WRIGHT

It is common among philosophers who take an interest in the phenomenon of vagueness in natural language not merely to acknowledge higher-order vagueness but to take its existence as a basic datum— so that views that lack the resources to account for it, or that put obstacles in the way, are regarded as deficient just on that score. My main purpose in what follows is to loosen the hold of this deeply misconceived idea. Higher-order vagueness is no basic datum but an illusion, fostered by misunderstandings of the nature of (ordinary, if you will 'first-order') vagueness itself. To see through the illusion is to take a step that is prerequisite for a correct understanding of vagueness, and for any satisfying dissolution of its attendant paradoxes.

I

The Ineradicability intuition

One standard motive for acknowledging higher-order vagueness is given exemplary expression by Michael Dummett:

> Now the vagueness of a vague predicate is ineradicable. Thus "hill" is a vague predicate, in that there is no definite line between hills and mountains. But we could not eliminate this vagueness by introducing a new predicate, say "eminence", to apply to those things which are neither definitely hills nor definitely mountains, since there would still remain things which were neither definitely hills nor definitely eminences, and so *ad infinitum* [sic]. [1]

This thought — the *ineradicability intuition* — may be generalised like this. Take any pair of concepts, F and G, with a vague mutual border. If you attempt to eradicate the vagueness by introducing a new term, H, to cover the shared borderline cases of F and G, your nemesis will be that the F-H and G-H borders will be vague in their turn. It follows, seemingly,[2] that the distinction between the Fs and the F-G borderline cases is itself *already* vague. Likewise for the Gs. So,

---

[1] From Dummett [1959], at p. 182 in Dummett [1978].

iterating, we have a hierarchy of levels of borderline cases of F, and another hierarchy of levels of borderline cases of G, each continuing indefinitely.

Notice how Dummett, like so many others, equates the lack of a sharp boundary between the Fs and the Gs with the (potential) existence of borderline cases, viewed as a *kind* of thing: things that are *neither definitely F nor definitely G*. I'll henceforward term this characterisation the Basic Formula. Moreover, Dummett does not, plausibly interpreted,[3] intend to allow that things which are neither definitely F nor definitely G might yet *be* F or G all the same— only just not definitely so. He is thinking of the kind in question as cases that in some way *come short* of being either F or G: if x is an 'eminence', then it fails to qualify either as a hill or as a mountain. So for there to be no definite line between hills and mountains is for there to be (potential) things 'in between' that are, in some way, of a *third sort*. Thus the mutual vagueness of F and G, on this view, consists in the existence of a certain kind of buffer zone between their respective (potential) extensions. Yet this buffer zone had better be blurry on both edges in turn, or F and G will turn out to be not mutually vague but sharply separated by a mutual neighbour. And now it seems we have no option but haplessly to allow the blurred buffer-zone model to reiterate indefinitely.

Dummett's thought is closely related to, though distinct in detail, from one of Russell:

> The fact is that all words are attributable without doubt over a certain area, but become questionable within a penumbra, outside of which they are again certainly not attributable. Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but unfortunately the penumbra itself is not accurately definable, and all the vaguenesses which apply to the primary uses of words apply also when we try to fix a limit to their indubitable applicability.[4]

Here Russell envisages not the introduction of a new term to apply within the "penumbra" but rather a moratorium on applying any term within it. If it is not certain that F is properly applied, then it is *not* to be applied — the penumbra is to be an exclusion zone. Still Russell's idea, like Dummett's, involves the notion of a *kind* of case separating those where the applications of F and not-F are

---

[2] It does follow, provided we assume that the introduction of the new term effects no alteration of the respective extensions of the original concepts; I'll come back to this point later.

[3] In "Wang's Paradox", he writes: "For, in connection with vague statements, the only possible meaning we could give to the word "true" is that of "definitely true"— (Dummett [1978], p. 256.) No doubt here are no borderline cases of "Definitely P" which are *clear* cases of P. The question is whether we should allow, as part of the intended meaning of the Definiteness operator, that it consists with something's being a borderline case of "Definitely P" that it yet be a case of P. Dummett is here saying no to that. We can call the principle that there are no truthful instances of the conjunctive form: P but not definitely P, *Dummett's Principle*. As will emerge later, there is actually considerable pressure against the principle.

[4] Russell [1923], at pp. 63-4 of the Keefe and Smith reprint.

respectively mandated, or "indubitable". And it is clear that he confidently expects judgments about membership in this kind to involve no less 'vaguenesses' than we started out with.

The ineradicability intuition impresses as highly plausible. The linguistic stipulations respectively envisaged by Dummett and Russell would indeed — surely — not have the effect of introducing precision. But can that really be enough to enforce the vertiginous hierarchy of borderline kinds?

II

The Seamlessness intuition

The ineradicability intuition is one motivation for postulating higher-order vagueness. A prima facie distinct motivation emerges from the idea that vagueness consists in the possession of borderline cases, together with one natural notion about how borderline cases, as characterised by the Basic Formula, come about.

Consider a case where, as many would allow, something akin to vagueness is induced by deliberate definitional insufficiency. Suppose we characterise the notion of a *pearl* as follows.[5]

(i) It is to be a sufficient condition for being a pearl that a candidate have a certain specified chemical constitution and appearance and be naturally produced within an oyster.

(ii) It is to be a necessary condition for being a pearl that a candidate have that same specified chemical constitution and appearance.

What about artificial pearls? They satisfy the specified necessary condition but not the specified sufficient one. One thing we might say is this: since there is no sufficient basis for classifying them either as pearls (for they do not satisfy the only specified sufficient condition) or as non-pearls (for they do satisfy the only specified necessary condition), it is so far *indeterminate* whether artificial pearls are pearls.[6] There is no fact of the matter.

---

[5] The example is John Foster's from classes in Oxford in the early 1970s. Compare Kit Fine's "nice_1" (Fine [1975] at p. 266), Timothy Williamson's "dommal", (Williamson [1990] at p. 107, and [1994] at pp. 213-4) and Mark Sainsbury's "child*" (Sainsbury [1991] at p. 173).

[6] One who, like Timothy Williamson, believes that Bivalence, like the Articles of the United States Constitution, is a self-evident truth, has of course to move differently: to deny that "pearl" has so far been endowed with a meaning, or — as proposed by Williamson himself — to regard artificial pearls as non-pearls purely by dint of their failure to satisfy any established sufficient condition for being pearls. See Williamson [1994] at p. 213 and [1997], section 3. ADD REFERENCE to Heck point

Now (this is the natural notion mentioned) suppose we think of borderline cases of naturally occurring vague predicates, —"bald", "heap", "red" and the other usual suspects—as relevantly like artificial pearls: cases which are left in classificatory limbo by a broadly analogous but naturally occurring kind of *semantic incompleteness*. Thus they are cases that do not meet any practice-established sufficient condition for satisfying the relevant predicate but do satisfy all practice-established necessary ones. This is, seemingly, a very intuitive way of thinking of the Basic Formula as being underwritten. The (definite) truths, and falsehoods, are what are determined as true, or false, by the facts and the semantic rules for the language in question. Borderline cases arise when the facts and semantic rules somehow fail to deliver.[7]

Next contrast the following two cases.

Case 1: You have a collection of 2-inch square colour patches, each of a uniform shade, collectively ranging in hue from red to orange, and numerous and varied enough to allow that every patch is matched by something that matches something in the collection that it does not match.[8] You have to arrange them in a 'monotonic' series; specifically, one such that the first patch is red and each subsequent patch is immediately preceded by something that is at least as red as it is. So your selection will consist in an initial batch of red patches followed by some which hover around the red-orange border followed by some orange ones, the whole giving the impression of a *perfectly seamless movement, without regression, from red to orange*.

Case 2: You have a collection of pearls, artificial pearls and costume (plastic) pearls and, again, have to arrange them in a monotonic series; specifically, a series such that the first selection is a pearl and each subsequent selection is immediately preceded by something whose case to be a pearl is at least as strong. Then your selection will consist in a string of pearls, followed by a string of artificial pearls, followed by the fakes.

---

[7] This type of view goes back to Frege and was for a long time regarded as datum, rather than theory. For modern exponents, see McGee and McLaughlin [1995], pp. 209ff; and Soames [2003] chapter 7, passim. For criticism, see Wright [2007] at pp. 419-423. Some of the criticisms there lodged are presented as depending on higher-order vagueness. I postpone to a future discussion the question whether they can survive in a qualified form if the conclusions of the present study are accepted.

[8] At least one commentator (Fara [2001]) has argued that this is impossible. I beg to differ — but the example could easily be reworked to finesse the issue.

The thought suggestive of higher-order vagueness is then simply this. Both series —we are currently supposing[9]— contain indeterminate cases, conceived as generated by semantic incompleteness. However, in the pearl series, the transitions from the pearls to the indeterminate cases, and from the latter to the non-pearls occur sharply, at specific places. And, associatedly, there is no second order indeterminacy—no indeterminacy in turn in the pearl-indeterminate and indeterminate-fake pearl distinctions. So, the thought occurs, how to explain the manifest difference in the phenomenology of the changes occurring within the two series if not by postulating second and, indeed, indefinitely higher-orders of indeterminacy in the red-to-orange series? How else to accommodate the fact that we are absolutely at a loss to identify specific first and last borderline cases of the red-orange distinction in that series, or indeed abrupt changes of any kind?

The key thoughts again: the vagueness of *pearl* and *red* is held to consist in the existence of borderline cases of these concepts, conceived as items that are not definitely classifiable as 'pearls', or as 'red', and not definitely classifiable as something else, on account of the semantic incompleteness of the relevant expressions. The sharpness of the distinction between the pearls and the borderline pearls shows in the abruptness of the transition between them in the relevant monotonic series. By contrast, the smoothness of the transitions between the reds and the borderline cases, and between the borderline cases and the oranges, enforces the idea that these distinctions are vague in turn. So it follows that they too admit of borderline cases. And so on *ad infinitum*.

We can call the driving intuition here the *seamlessness* intuition.[10] In general: unless we have an indefinite hierarchy of kinds of borderline case, it seems there will have to be sharp boundaries in any process of transition between instances of one vague concept and instances of another. Or so it anyway appears. But we'll return to explore this thought in some detail.

The resulting broad conception of full-blown higher-order vagueness: the conception of an infinite hierarchy of kinds, each potentially serving to provide an exclusion zone and thereby prevent a sharp transition, in a suitable series, between instances of distinctions exemplified at the immediately preceding stage of the hierarchy, may be termed the *Buffering view*. I shall argue for each of the following claims:

---

[9] In case it is not obvious, I do *not* think that this is the right way to conceive of the vagueness of the "usual suspects".

[10] I prefer "seamlessness" to "continuity". The relevant notion is pre-mathematical and intuitive. Compare Fara [2004].

(i) That the Buffering view is not well motivated by either the Ineradicability or the Seamlessness intuitions.

(ii) That there is serious cause to question whether the Buffering view is fit for purpose.

(iii) That for the kinds of vague concepts—the "usual suspects"— in which we are interested, the view that they exhibit higher-order vagueness on the model of the Buffering view is at odds with the broadly correct conception of their ('first-order') vagueness.

III

Potential confusions about higher-order vagueness — three distinct notions

Of course, philosophers are free to mean by the phrase, "higher-order vagueness", whatever they choose. But the fact is that at least three distinct putative phenomena have been earmarked by it in the literature, without — perhaps — all of those who have so earmarked them being clear that their discussions concerned potentially different things. One is:

> (a) That the distinction between the things to which a vague expression applies and its first-order borderline cases —the cases where it is indeterminate whether it or its complement applies— does itself, in the cases that characteristically interest us, admit of borderline cases; that the distinction between the things to which a vague expression applies and this *second-order* of borderline cases also admits of borderline cases; that the distinction between the things to which a vague expression applies and this *third-order* of borderline cases also admits of borderline cases; and so on indefinitely. When, in the fashion noted, borderline cases are thought of as an intermediate *kind*, distinguished from the kinds of which they are borderline cases, this idea becomes **the Buffering view**.

Standing apparently unrelated to that is

> (b) **The vagueness of *Vague***: there are concepts which are borderline cases of the *vague-precise* distinction itself, — concepts which are neither definitely vague nor definitely precise, — and, further, there are borderline cases of membership of this range of concepts in turn, and borderline cases of those in turn……. and so on. [11]

Then finally there is the thought

> (c) That the usual kind of definiteness operator —that is: one introduced for the purpose of allowing us to characterise the borderline cases of F in accordance with the Basic Formula — ineluctably gives rise to a hierarchy of new, pairwise inequivalent vague expressions, "Definitely F", "Definitely Definitely F" and the like.[12] (**Definitisation modifies truth-conditions but does not eliminate vagueness.**)

---

[11] This discussion seems to originate in Sorensen [1985]. See Hyde [1994] and [2003], and Varzi [2003].

[12] See, for example, Williamson [1999].

It seems obvious enough that there is little connection between (b) and the other two. It seems quite consistent with holding to the Buffering view, or with thinking of "Definitely P" as vagueness-inheriting though precision-increasing when applied to a vague claim P, that the notion of vagueness itself should divide all expressions into two sharply bounded kinds—that there is never any vagueness about the question whether an expression is vague or not. Conversely, one might think of the distinction between vague expressions and others as admitting of borderline cases but hold to a view of the nature of vagueness according to which there are no higher-order borderline cases; and one might simultaneously repudiate any operator of definiteness, or take the view that any legitimate such operator generates only precise claims. At any rate, these are all prima facie compatibilities. If there are deeper tensions, that would be interesting — but they remain to be brought out.

I will say nothing further here about thesis (b). Of potentially more importance for our purposes is the apparent distinctness of thesis (a) and thesis (c), the thesis that applications of the Definiteness operator, while they shift truth-conditions (since they take any originally indefinite claim to a false one), are nevertheless impotent to eliminate vagueness: if P is vague, so is Definitely P. Thesis (a) takes the distinction between F and (any order of) its borderline cases to be vague. F's higher-order vagueness consists, at each nth order, n >1, in the (potential) existence of borderline cases of the distinction between F and its borderline cases of the immediately preceding order. The thought embodied by thesis (c), by contrast, changes the terms of the relation of mutual vagueness. At second-order, for example, it is not F but "Definitely F" that is assigned a vague borderline. More specifically, letting '*Def*' be the definiteness operator, the 'second order' of borderline cases countenanced by thesis (c) may be schematised thus:

$\sim Def Def\text{F} \,\&\, \sim Def(\sim Def\text{F} \,\&\, \sim Def\sim\text{F})$

And in general each successive nth order of vagueness, n > 1, is conceived as consisting in the vagueness of the boundary between the Def_{n-1}Fs — the things that are definitely….definitely (n-1 times) F — and the definite borderline cases of order n-1, that is, as consisting in the (potential) existence of cases satisfying the condition:

$\sim Def_{n}\text{F} \,\&\, \sim Def(\text{Borderline}_{n-1}\text{F})$

Now, as a construal of the notion of higher-order vagueness as suggested by the ineradicability and seamlessness intuitions, this initially just seems wayward. Those intuitions motivate a thesis about the existence of a hierarchy of orders of vagueness of a single originally

targeted concept. Thesis (c) by contrast goes in for a hierarchy of kinds of *first-order* vagueness which successively concern *different* concepts: *Definitely F*, *Definitely Definitely F*, … and so on, — a hierarchy produced as an artifact of the introduction of the Definiteness operator. The preoccupation of much of the discussion with thesis (c) might therefore seem to offer one more example of philosophers taking their collective eye off the ball. It is hardly intuitively evident that natural language contains any operator that behaves like this. And even if it does, what can that have to do with the proper understanding of the nature of vagueness, which presumably comes fully formed, as it were, — and therefore fully 'higher-orderised', if the phenomenon is indeed real, — even in languages lacking any Definiteness operator? Aspects of the behaviour of such an operator cannot constitute higher-order vagueness as originally motivated. What does thesis (c) have to do with anything?

Here is one arguable connection. When the first-order borderline cases of the distinction between F and its negation are characterised by the Basic Formula, they will be, one and all, things that are not definitely F. So they will fall under the negation of "definitely F" and will thus, none of them, be borderline-cases of "definitely F".[13] Now thesis (a) requires that there are borderline cases of the distinction between F and its first-order borderline cases. These will all, presumably, be clear cases of "not definitely not F". So if they are borderline cases of the Basic Formula's characteristic conjunction, they must be borderline cases of "not definitely F". But if they were definite cases of "definitely F", they would not be borderline cases of its negation. So they must be borderline cases of "definitely F" too, which is therefore vague if thesis (a) is true of F and borderline cases are characterised by the Basic Formula.

However thesis (c) involves two components: that definitisation does not eliminate vagueness, just argued for, and that it generates statements which are not, in general, equivalent to those definitised. Since it is, intuitively understood, a factive operation, the second component is tantamount to the claim that a definitised statement is in general logically stronger than its prejacent. This too is, as will emerge, plausibly taken to be a consequence of thesis (a) and the characterisation of borderline cases given by the Basic Formula.

What about the converse direction? Is thesis (a) a consequence of thesis (c), assuming the Basic Formula? Again, arguably so. Let G be any predicate such that the F-G distinction is vague.

---

[13] This step, *nota bene*, applies Dummett's Principle. See note 3 above.

Then F has borderline cases, characterised as cases which are not definitely F and not definitely G. But by thesis (c), "definitely F" is vague if F is. And, since by hypothesis G is vague, so is "definitely G". Since vagueness is, presumably, preserved under negation, "not definitely F" and "not definitely G" are likewise vague. Since vagueness is presumably preserved under (consistent) conjunction, so is "not definitely F and not definitely G" — so the notion of a borderline case of F is itself vague, and hence has borderline cases. These cannot be definite cases of F or they would fail the first conjunct and hence not be borderline cases of the conjunction. So they must be borderline cases of F and of the notion: *borderline case of F and G*. The latter notion is then available for choice in place of 'G', and the reasoning can be iterated indefinitely.

So, given that the vagueness of a predicate consists in its susceptibility to borderline cases and the thesis that these are one and all to be characterised as per the Basic Formula, there is a case — we can put it no stronger than that — that thesis (a) and thesis (c) are equivalent. If that is right, it offsets the charge of irrelevance against intended investigations of higher-order vagueness that have taken thesis (c) to be a constitutive matter. On the other hand, if thesis (a) depicts an illusion, the equivalence will mean that the illusion persists in thesis (c) as well. Work on the semantics and proof-theory of the definiteness operator directed towards the elucidation and stabilisation of thesis (c) will then be so much misdirected effort.

IV

The Basic Formula and Lack of Sharp Boundaries

So let's assume for the sake of argument that borderline cases are felicitously described by the Basic Formula, and — thesis (a) — that certain concepts sustain an infinitely ascending hierarchy of orders of borderline case, each characterisable by a suitable application of the Basic Formula. What reason is there, in this setting, to think that the Definiteness operator should comply with the proof theoretic part of thesis (c): the claim that definitisation increases logical strength?

In fact there is quite powerful pressure towards that thought. It comes from reflection on that form of the Sorites paradox — what I once called the No-Sharp-Boundaries paradox — which seems to connect most directly with the very nature of vagueness.[14] I'll make the point in some detail over this and the succeeding section.

---

[14] Wright [1987]

The standard form of major premise for the Sorites is a universally quantified conditional, usually motivated by tolerance intuitions. But the major premise for the No-Sharp-Boundaries paradox takes the form of a negative existential,

(i) $\sim(\exists x)(Fx \ \& \sim Fx')$,

seemingly tantamount merely to the affirmation that F is indeed vague in the series in question. For vagueness is just the complement of precision, and precision (relative to the relevant kind of series) is, it seems, perfectly captured by

$(\exists x)(Fx \ \& \sim Fx')$.

But whereas it may be doubted that vague predicates really are tolerant, it hardly seems doubtful that they really are vague! In affirming (i), accordingly, we seem merely to have affirmed that F is vague.[15] So vagueness appears paradoxical *per se*.

Enter the Definiteness operator. What, it may be suggested, really constitutes precision is a sharp boundary between *definite* cases. Hence what is really tantamount to an expression of F's vagueness in the relevant series is not the negative existential statement (i) above but rather:

(ii) $\sim(\exists x)(\mathit{Def}Fx \ \& \ \mathit{Def}\sim Fx')$

---

[15] We obtain a sorites paradox from the negative existential major premise without reliance on any distinctively classical moves, by running right-to-left, as it were— by beginning with a minor premise of the form, ~Fa, and reasoning through successive steps via the rules for conjunction, existential introduction and the (intuitionistically acceptable) negation-introduction half of *reductio*.

It merits emphasis that the intuitive motivation for the major premises for Sorites paradoxes varies quite dramatically across forms that are classically equivalent. Consider for instance the three genres of premise:

(i) $(\forall x)(\sim Fx \ V \ Fx')$
(ii) $(\forall x)(Fx \rightarrow Fx')$
(iii) $\sim(\exists x)(Fx \ \& \sim Fx')$

The last, as noted, is naturally motivated just by the thought that it is constitutive of the vagueness of a predicate that its extension in a suitably constructed series of objects not run right up against that of its negation. This thought involves no intuitive dependence on Bivalence. The second is driven, more specifically, by tolerance intuitions, of the kind discussed in Wright [1975], that in turn draw on folk-semantical ideas about observational and phenomenal predicates which have little explicit connection with vagueness. These ideas, again, involve no intuitive dependence on Bivalence but are stronger than the thought that motivates (iii) since someone who embraced a 'Third Possibility' view of borderline cases could accept (iii) while rejecting (ii): vagueness might be conceived as, in typical cases, *intolerant* of the distinction between some Fs and some borderline cases of F, even though sustaining no-sharp-boundaries principles in the form of (iii). (i), finally, is entailed by either of the other two if, but only if, Bivalence is assumed for predications of F.

It is thus natural to conceive of (i) through (iii) as of decreasing strength. It is a significant weakness of the classical outlook that it stifles their intuitive differences.

— the thesis that there is no last *definite* case of F in the series immediately followed by a first *definite* non-F. But (ii), unlike (i), gives rise to no immediate paradox. We can show of course by appeal to it that any n such that *Def*~Fn', must be such that ~*Def*Fn. But then — absent further proof-theoretic resources for the Definiteness operator— we seem to have no means to commute the occurrences of '~' and '*Def*' to generate something soritical.

What, though, —other than the reflection that we can apparently finesse the paradox thereby— is available to justify the claim that it is indeed (ii), rather than (i), that gives proper expression to F's vagueness in the kind of series in question?

There is a very good argument for that claim if we can legitimately have full recourse to classical logic. Take it that what F's vagueness in the series consists in is the presence there of (first order) borderline cases of F, and that these are suitably characterised by the Basic Formula. Specifically, suppose that there is such a borderline case of F:

(iii) $(\exists x)(\sim Def Fx \ \& \ \sim Def \sim Fx)$

but also, for *reductio*, that there *is* a last definite case of F in the series immediately followed by a first definite non-F:

(iv) $(\exists x)(Def Fx \ \& \ Def \sim Fx')$

Contradiction follows on the assumption of the *monotonicity* of the series (intuitively, that all the F-relevant changes manifested in it are one-directional), which we may capture by the pair of principles:

$$(\forall x)(Def Fx' \rightarrow Def Fx)$$

— the immediate predecessor of anything definitely F is definitely F —

and $(\forall x)(Def \sim Fx \rightarrow Def \sim Fx')$

—the immediate successor of anything that is definitely not F is likewise definitely not F. For suppose m is a witness of (iv); that is,

$$Def Fm \ \& \ Def \sim Fm',$$

Then the monotonicity principles will ensure that every element preceding m in the series is Definitely F and every element succeeding m' is Definitely not F; and hence that none satisfies the rubric for borderline cases given by the Basic Formula, contrary to (iii).

We supposed that the vagueness of F in the series in question consists in the presence of borderline cases of F, as characterised by the Basic Formula. The preceding reasoning establishes that one who accepts that supposition thereby commits themselves to (ii). So in order to show that it is (ii), not the soritical (i), that is tantamount to an acceptance that F is vague in the series in question, we now require the converse direction: that someone who accepts that there is no last definite F element immediately succeeded by a first definite non-F element is thereby committed to the existence of borderline cases of F in the series concerned, as characterised by the Basic Formula. Straightforward — though classical —reasoning establishes the point. The series, we can take it, is such that

(1) *Def*(F0)

and (2) *Def*~(Fn)

Suppose (ii) and for *reductio* the negation of (iii):

(3) ~(∃x)(~*Def*Fx & ~*Def*~Fx), —there are no borderline cases of F in the series.

Then (4) *Def*~Fx' —> ~*Def*Fx, — from (ii).

So (5) ~*Def*(Fn-1), — from (2) and (4).

Suppose (6) ~*Def*~(Fn-1)

Then (7) (∃x)(~*Def*Fx & ~*Def*~Fx), —contrary to 3.

So (8) (~~)Def~(Fn-1).

…….

This routine may be repeated eventually culminating in contradiction of 1. At that point (3) may be discharged by *reductio*, on (1), (2) and (ii) as remaining assumptions, a final step of double negation elimination then yielding (iii).

Our result, then, is that — granted classical logic — F's vagueness, identified with its possession of borderline cases as characterised by the Basic Formula, is equivalent not to the soritical

(i) ~(∃x)(Fx & ~Fx'),

but the apparently harmless

(ii) ~(∃x)(*Def*Fx & *Def*~Fx').

It is the latter, then, which, we may accordingly be encouraged to think, is the canonical expression of F's lack of sharp boundaries in the relevant kind of series.

This result is the first point towards uncovering the advertised impetus towards the proof-theoretic component of thesis (c). I will pursue that further in the next section. It may also seem (as it once did to me) to be the first step towards a dissolution of the No-Sharp-Boundaries paradox. Obviously, however, it is at most a first step. For one thing, the reliance on classical logic is, of course, of some moment in this context. The question under review is whether, and if so, how a correct understanding of the nature of vagueness escapes a commitment to a soritical version, such as (i), of the No-Sharp Boundaries intuition. In exploring the matter, we therefore must resort only to principles of inference which are sound for vague languages. Those who share the doubt of the present author whether classical logic is in that case should therefore regard the reasoning just run through with at most qualified enthusiasm.

Even were we satisfied that classical logic is fit for duty in this setting, however, there is a further issue. For unless we are prepared to allow that the boundary between the definite Fs and the borderline cases of F is sharp, there is the same motivation as previously to affirm

(i)* $\sim(\exists x)(\mathit{Def}Fx \,\&\, \sim\mathit{Def}Fx')$,

and this, if allowed, will in turn subserve a Sorites paradox (this time subverting the distinction between the borderline cases and the definite cases of F.) To be sure, the reply can be that the proper way to do justice to the vagueness of the second-order borderline is to affirm not (i)* but

(ii)* $\sim(\exists x)(\mathit{DefDef}Fx \,\&\, \mathit{Def}\sim\mathit{Def}Fx')$

—there is no sharp cut-off separating the *definite* cases of 'Definitely F' from the *definite* borderline cases of F. And in general, for an arbitrary pair of mutually vague, contrary concepts, ϕ and ψ, manifested in the series in question, it may be proposed, generalising the reasoning above, that the proper way to give expression to a lack of sharp boundaries between them is to affirm the negative existential,

(*) $\sim(\exists x)(\mathit{Def}\phi x \,\&\, \mathit{Def}\psi x')$

So we need never, apparently, be committed at any level to a soritical claim.

But where is this leading? If the seamlessness intuition is to be upheld, then it seems that it must be possible, in principle, so to describe a Sorites series that no abrupt transitions of any relevant

kind take place between adjacent elements within it. So *every* pair of contrary concepts, ϕ and ψ, manifested in the series must sustain the truth in it of the relevant instance of (*). More specifically: if the mutual vagueness of any pair of concepts, *Def*(…x…) and *Def*~(…x…), is viewed as consisting in the existence of borderline cases as characterised by the Basic Formula, and if the seamlessness intuition is accepted, then we are committed to each of the following principles:

~(∃x)(*Def*Fx & *Def*~Fx')

~(∃x)(*DefDef*Fx & *Def*~*Def*Fx')

~(∃x)(*DefDefDef*Fx & *Def*~*DefDef*Fx')

...etc.

Given the reliance on classical logic of the reasoning worked through above, it would be tendentious to proclaim these *Gap principles*[16] to be respectively *characteristic* of the putative successively higher-orders of borderline case of the predicate F. But they are at least, it seems, among our commitments if we accept that a series is possible in which a seamless, monotonic transition is effected from instances of F to instances of not-F, and in which any borderline cases of any distinction exemplified within it are characterised by the Basic Formula as applied to that distinction.

Let us take stock. It is hard to reject the idea that the seamlessness intuition is sound in some form: the transition from Fs to non-Fs in a Sorites series can be effected without abrupt, noticeable change of status at any point. The thought that leads from Seamlessness to the postulation of higher-order vagueness can be refined as follows. Define a monadic predicate (open sentence) as *F-relevant* if it is formulated using just F, the truth functional connectives and the definiteness operator. Conceive of seamless transition as the circumstance that the ranges of each pair of incompatible F-relevant predicates exemplified in a Sorites series running from instances of F to instances of its negation are *buffered*: between the instances of any such pair of predicates intervenes at least one element to which neither definitely applies: an element which is a borderline case of the distinction they express, according to the characterisation of borderline cases given by the Basic Formula. As we saw, this conception, assuming monotonicity in the transition concerned, ensures that a Gap principle — an instance of (*) — holds for any such pair of predicates. On classical assumptions, the holding

---

16 Delia Fara's nice term in her [2004]. Each such principle (Fara actually formulates them slightly differently) classically ensures that the instances in a suitable series of a pair of contrary concepts of the form, Def_nϕx and $Def\sim Def_{n-1}$ϕx, are separated by a gap — in our terminology above, a buffer zone.

of such a Gap principle is *equivalent* to the presence in the series of a borderline case, characterised as per the Basic Formula, of the original distinction. So the train of thought is this:

- Seamlessness requires buffering of all F-relevant distinctions exemplified in the series;
- Such buffering requires the presence, in the series, of borderline cases (characterised as per the Basic Formula) of each such distinction;
- The presence of such borderline cases requires (indeed, classically, is tantamount to) the holding of appropriate Gap principles.

That said, though, note that a plausible connection between Seamlessness and the Gap principles can of course be made out more directly. If *any* of the existential statements which the Gap principles respectively directly contradict is true in a Sorites series, then there is an abrupt, non-seamless change of status between the element that witnesses that statement's truth and its immediate successor. So seamlessness, it appears, requires the Gap principles to hold anyway, whether or not we take that to be equivalent, as classically it is, to the presence of borderline cases of each appropriate higher order.[17]

V

Thesis (c) and the Paradox of Higher-order vagueness

Let us now connect the foregoing with the proof-theoretic component of thesis (c). I once argued that, so far from resolving the No-Sharp-Boundaries paradox, to corral our no-sharp-boundaries intuitions into an endorsement of principles of the (*)-form merely generates new soritical problems.[18] The argument utilised a proof-theory incorporating the rule:

---

[17] Note that anyone content with classical logic in this region who accepts the idea that seamless transition is possible and that it is correctly construed as requiring the Gap Principles to hold *en masse*, should worry about this: that no *finite* sorites series can exemplify borderline cases of every higher order unless some borderline cases instantiate multiple, indeed infinitely many orders. (This is noted in Fara [2004] at p. 205.) Given the ways, reviewed earlier, in which acceptance of higher order vagueness is standardly motivated, this — egregious violation of Dummett's principle — is an idea for which we are wholly unprepared, indeed an idea of questionable intelligibility.

[18] Wright [1992]. The argument was there presented as a *reductio* of the very idea of higher-order vagueness. In fact, what it puts under pressure is any set of assumptions entailing an nth-order Gap Principle, $n > 1$. The picture of higher order vagueness captured by the Buffering view incorporates one such set of assumptions, as we have seen. But we have also noted that the very idea of seamless transition appears to enforce the Gap Principles as well.

Focused on the case second-order Gap Principle, presumed itself to be a Definite truth, the argument was this:

(DEF) $\underline{\{A_1 \ldots A_n\} \Rightarrow P}$
$\{A_1 \ldots A_n\} \Rightarrow \mathit{Def}P$,

where $\{A_1 \ldots A_n\}$ contains only 'fully definitised' propositions (i.e., propositions prefixed by '*Def*'.)

Once *Def*'s proof-theory incorporates this rule, each of the Gap principles corresponding to the successive higher orders of vagueness becomes soritical.[19] But the Gap principles, as we have seen, are seemingly imposed by the possibility of seamless transition across a sorites series. Moreover, classically, each is tantamount to — and each is anyway a consequence of — an affirmation of the existence of a corresponding order of borderline cases, when characterised in accordance with the Basic Formula. So the postulation of any higher order of borderline cases is soritical unless the DEF-rule fails. And if seamless transition does indeed entail the Gap principles, then — even without classical logic — we must likewise accept that the DEF-rule fails provided we believe that seamless transition is possible.[20]

To reject the DEF rule is to allow that D*ef*P can be a consequence of a set of (fully definitised) premises, even though *DefDef*P is not. Since the entailment from *DefDef*P to *Def*P is unquestioned, to reject the DEF-rule is thus to regard the definitisation of a sentence as potentially increasing its logical strength. That is the proof-theoretic component of thesis (c).

(n.18 cont.)

1	(1)	*Def*~(∃x)[*Def*(*Def*(Fx)) & *Def*(~*Def*(Fx´))]	Assumption
2	(2)	*Def*(~*Def*(Fk´))	Assumption
3	(3)	*Def*(Fk)	Assumption
3	(4)	*Def*(*Def*(Fk))	3, DEF.
2,3	(5)	(∃x)[*Def*(*Def*(Fx)) & *Def*(~*Def*(Fx´))]	2, 4, ∃-intro.
1	(6)	~(∃x)[*Def*(*Def*(Fx)) & *Def*(~*Def*(Fx´))]	1, *Def*-elim.
1,2	(7)	~*Def*(Fk)	3, 5, 6, Reductio
1,2	(8)	*Def*(~*Def*(Fk))	7, DEF
1	(9)	*Def*(~*Def*(Fk´)) -> (*Def*(~*Def*(Fk))	2, 8 Conditional Proof

[19] See the proof schema illustrated in note 17. Note that the general applicability of the schema assumes, in addition, that the Gap Principles are definite truths, and that there are definite borderline cases of the relevant order. These points would need defence in a fully rigorous presentation of the line of thought currently under development.

[20] I am passing over some subtleties here – see note 21.

VI

A revenge problem for the Buffering view

Let's review the dialectic to this point. In the cases that interest us (the "usual suspects"), it is not, claimed Dummett and Russell, possible to eliminate vagueness by annexing a new expression to the borderline cases of a distinction, since the distinctions between items to which the new expression applies and those that fall under either of the original concepts will remain vague. However it is typically possible so to arrange the elements of a soritical series for a concept ϕ that an apparently seamless transition is effected from instances of it to instances of some contrary concept, where seamlessness involves that no salient, relevant changes occur between any element of the series and its successor. Higher-order vagueness is meant to provide a natural and plausible explanation of both these putative items of data. Annexure of a new expression to the borderline cases of a distinction never results in precision because the concept to which the term is thereby annexed is itself a vague concept in it own right. Seamless transition is possible because it is possible so to engineer a soritical series that every pair of contrary concepts manifested within it are buffered by borderline cases of their contrast. This in turn requires the failure of the DEF rule, if sorites paradoxes are not to recur. Where P is vague, *DefDef*P must in general be logically stronger than *Def*P, although still vague.[21]

There are a number of issues on which a fully satisfactory development of the Buffering view would have to elaborate. Three in particular are especially salient. First, it will not do, obviously, just to reject the DEF rule on the grounds that paradox will otherwise be reinstated. Rather, a semantics is wanted for the Definiteness operator to underwrite the failure of the rule and explain more generally what form an appropriate proof-theory for the operator should assume. Second, any genuinely explanatory such semantics had better be grounded in further insight into the nature of borderline cases — an insight which somehow explains *why* the borderline cases of any vague distinction are themselves a vaguely demarcated kind. Third, it needs to explained how exactly a finite sorites series can indeed provide for a seamless transition between incompatible descriptions. It is not enough to gesture at the idea of buffering by borderline cases: we need to be told in detail how a seamless transition may be fully adequately described.[22]

---

[21] Let me offer just a quick outline of some of the "subtleties" around this point advertised in n. 20…….

[22] This problem — what Mark Sainsbury christened the Transition Question (1992) —for an adequate account of vagueness has not been reflected in the degree of attention to it meted out in the literature. It is

I do not believe that the Buffering view can deliver on these obligations. I shall not here, however, further consider what might be done to address the first.[23] For the second, the notion that the borderline cases of a vague distinction constitute a further vague kind taking a place, so to speak, in the same broad space of possibilities as the poles of that distinction, — this notion is exactly the illusion that I aim to expose. The third issue — the Transition Problem — will occupy us in the next section. The task for this section is to table an argument that, even before any further development is attempted, the Buffering view is susceptible to a new paradox.

The paradox is a kind of 'revenge' problem, consequent on the possibility—as it appears—of defining a distinct operator of *absoluteness* in terms of that of definiteness as follows:

*Abs*P is true if and only if each Def_nP is true for arbitrary finite n.

There seems no reason to contest that such an operator is well defined if *Def* is, nor that, intuitively, it should have some actual cases of application. Consider, for instance, Kojak, a man microscopic examination of whose scalp —under whatever degree of magnification—reveals no distinction, in point of the presence of hair fibres, from the surface of a billiard-ball. Does it make any sense to suppose that any of

Def[Kojak is bald], Def_2[Kojak is bald], Def_3[Kojak is bald]…. Def_n[Kojak is bald],……

fails of truth or is somehow less acceptable than a predecessor in the series?

By its definition, *Abs*P entails *Def*P; so in particular any statement of the form *Abs*(At) entails *Def*(At), and therefore any statement of the form (∃x)(*Abs*Ax) entails the corresponding (∃x)(*Def*Ax). Contraposing, any statement of the form, ~(∃x)(*Def*Ax) entails the corresponding ~(∃x)(*Abs*Ax). Since any Gap principle for definiteness is —assuming that *Def* distributes across conjunction and collects conjuncts in the obvious way—equivalent to something of the former form, acceptance of any Gap principle for definiteness is a commitment to acceptance of the corresponding Gap principle for absoluteness.

---

in effect the issue raised by the Forced March sorites: the problem of explaining how a competent subject who is charged to give nothing but correct, maximally informative verdicts may respond, case by case, to the successive members of a soritical series without at any point committing himself to some kind of abrupt (and incredible) threshold. If the Buffering view can genuinely provide an account of seamless transition, it will provide the descriptive resources that the hapless subject of the Forced March needs. I shall pour cold water on the prospects later.

[23] For development of some misgivings about the ability of supervaluational approaches, at least, to deliver on this aspect, see Fara [2004].

That is all as intuitively it should be. But now observe that, whatever the position with *Def*, the absoluteness operator, so defined, should be iterative.[24] So the effect, just provided that the relevant Gap principle is itself absolute, and that the relevant polar verdicts are assumed absolute, is to reintroduce a version of the No-Sharp Boundaries paradox. The proof is just the obvious adaptation:

1	(1)	*Abs*~(∃x)[*AbsAbs*(Fx) & *Abs*~*Abs*(Fx´)]	Assumption —absoluteness of 2nd order Gap principle for *Abs*
2	(2)	*Abs*~*Abs*(Fk´)	Assumption of polar absoluteness
3	(3)	*Abs*(Fk)	Assumption for reductio
3	(4)	*AbsAbs*(Fk)	((3), iterativity of *Abs*
2,3	(5)	(∃x)(*AbsAbs*(Fx)) & *Abs*~*Abs*(Fx´))	(2),(4), ∃-intro.
1	(6)	~(∃x)(*AbsAbs*(Fx)) & *Abs*~*Abs*(Fx´))	(1), *Abs*-elim.
1,2	(7)	~*Abs*(Fk)	3,5,6, RAA.
1,2	(8)	*Abs*~*Abs*(Fk)	7, iterativity and closure for *Abs*
1	(9)	*Abs*~*Abs*(Fk´) -> *Abs*~*Abs*(Fk)	2,8 CP.

---

[24] This excellent observation is due to Elia Zardini. Here is a sketch of one plausible demonstration of it:

1	(i)	*Abs*A	Assumption
1	(ii)	*Def*A & *DefDef*A & …….	(i) Definition of *Abs*
1	(ii)	*DefDef*A & *DefDefDef*A & …..	(ii) &E
1	(iv)	*Def*(*Def*A & *DefDef*A & …….)	(iii) collection for *Def* over conjunction
1	(v)	*DefAbs*A	(iv) Definition of *Abs*
	(vi)	*Abs*A —> *DefAbs*A	(i), (v) Conditional Proof
	(vii)	*Def*(*Abs*A —> *DefAbs*A)	(vi) *Def* Intro — see below*
	(viii)	*DefAbs*A —> *DefDefAbs*A	(vii) Closure of *Def* over entailment
1	(ix)	*DefDefAbs*A	(v), (viii), MPP
	(x)	*Abs*A —> *DefDefAbs*A	(i), (ix) Conditional Proof

…….

and so on. Thus each $Def_n Abs$A can be established on *Abs*A as assumption. *AbsAbs*A is accordingly a semantic consequence of *Abs*A.

* The principle appealed to is that if |=A, then |=*Def*A. This should be uncontroversial—presumably all necessary truths are definite.

In sum: The Gap principles may or may not be directly soritical when augmented by whatever may prove to be the appropriate proof-theory for *Def*. But even if they are not, there seems no objection to introducing the *Abs* operator as defined, if there is no objection to *Def* in the first place. If as argued, *Abs* is iterative, and if it is an absolute truth that a (first order) borderline case of F is not an absolute case of F, and if the Gap principles for *Def* are absolute truths (whence those for *Abs* are also), then the Gap principles for *Def* do ultimately spawn a sorites paradox in any case, even if they are innocent of paradox when worked on merely via the appropriate proof theory for *Def*.

VII

The Transition Problem

No doubt, there are lines of resistance for the defender of Gap principles to explore.[25] But we must delay no further in attending to a more basic difficulty which has been shadowing the discussion all along and is in the end, I suggest, decisive that the attempt to capture the seamlessness intuition by means of an apparatus of ascending Gap principles, a fortiori by means of limitless Buffering,[26] is fundamentally misconceived.

Let's step back. The seamlessness intuition, as interpreted by the Buffering view, has it that in any Sorites series for a concept F, no pair of adjacent elements are characterised by incompatible F-relevant predicates.[27] Somehow a seamless transition is effected from (Definite$_n$) Fs at one end to (Definite$_n$) non-Fs at the other. The move to an apparatus of Gap principles is a response to this thought which interprets it as requiring that every incompatible pair of predicates, Φ and Ψ, formulable using just F, *Def* and negation, which are exemplified in the series must be buffered—there have to be intermediate elements whose strongest F-relevant characterization is compatible with both Φ and Ψ. These are the borderline cases of the Φ-Ψ distinction.

One direct corollary of this way of handling seamlessness which it is time—rather belatedly—to take proper note of is that if the Basic Formula is to offer a viable characterization of

---

[25] One is to query the status of the minor premises. To treat the reasoning outlined as a Sorites paradox, properly so termed, requires that its conclusion — *Abs*~*Abs*(F0) — confounds an acceptable such premise. Indeed it does if F(0) is absolutely true. But if F(0) were, say, merely definitely true (!), might that not be consistent with its also being an absolute truth that it is not absolutely true? For considerations in this direction, see Williamson [1997a] and Dorr [2009].

[26] Which, recall, is classically the same thing.

[27] Recall that a predicate is *F-relevant* if it is formulated using just F, negation, conjunction and the definiteness operator.

borderline cases, we *have* to think of "~*Def*Φx & ~*Def*Ψx" as compatible with both Φx and Ψx. So "Φx & ~*Def* Φx" has to be a consistent description; and hence, it appears, we have after all to take seriously the possibility that there are items which satisfy it —things which while being a certain way, are not definitely that way. Dummett's Principle has to be jettisoned if the Buffering View is to have any chance of delivering seamlessness. And with it goes any Third Possibility interpretation of borderline status.

If the rejection of Dummett's Principle seems like nonsense, we might try to discount that impression as owing to the intrusion of inappropriate resonances associated with the English word "definitely". We are after all, it may be said, introducing a term of art for certain theoretical purposes. But that would be a pretty rich response, given that it was exactly the resonances of the natural language word that made the Basic Formula seem apt in the first place.

Be that as it may, the basic problem remains, even after Dummett's Principle is surrendered, that the idea of limitless buffering in accordance with the Basic Formula, rather than providing for a lucid understanding of the possibility of seamless transition, seems, when pressed, merely to plunge into aporia. The difficulty is best elicited in the context of a version of the Forced March. Suppose you are the subject and that you have returned a correct verdict —Φ — concerning element m. If Φ and ~*Def* Φ are compatible, then you now have the option of describing m' as an instance of the latter without explicit concession of a change in Φ -relevant status. Well and good. Nevertheless since *Def* is factive, some elements correctly describable as ~*Def*Φ will be so because they are Ψ. And m' had better not be one of those, or the transition from m to m' will mark a sharp boundary in the series after all. On the other hand, if m' *is* also Φ, — as compatibly with its correct description as ~*Def* Φ it may now be —then the buffer zone is merely narrowed by one element and we can push on to m" and raise the same possibilities again: is m" an instance of ~*Def*Φ because it is Ψ?— in which case there is a sharp boundary — or is it also an instance of Φ? — in which case the buffer zone narrows again. Obviously, the buffer zone must not narrow too far, or there will be a sharp cut-off between Φ and Ψ in any case. So it appears that we have to think in terms of there being cases which are correctly describable as ~*Def* Φ but not because they are Ψ, and which also —if narrowing of the buffer zone is to be halted —do not exploit the compatibility of Φ and ~*Def* Φ by being Φ. These cases will constitute a distinctive kind of borderline case between Φ and Ψ: cases that qualify for characterisation in terms of the Basic Formula without exploiting the compatibility, after the surrender of Dummett's Principle, of Φ with ~*Def* Φ and of Ψ with ~*Def* Ψ. It is essential

that such cases occur if a seamless transition is to be effected. For if they do not, each case within the region characterised as ~*Def* Φ and ~*Def* Ψ, will either be Φ and or Ψ. So to solve the transition problem, we need to provide you — the subject — with the means in principle, whatever epistemological difficulties you might encounter in practice, to mark the occurrence of such cases. But how can that be done?

This is already a fatal objection to the prospects for solving the Transition Problem using the resources at hand, since we now appear to be committed to recognising a kind of indeterminacy for which the apparatus of Φ-relevant and Ψ-relevant predicates and the Basic Formula provides no adequate means of expression— cases whose description in accordance with the Basic Formula *masks their distinction* from others which it also characterises but which are, so to say, tacitly polar. There is therefore no prospect of doing justice to seamless transition using just the notion of buffering by borderline cases, conceived in accordance with the Basic Formula. But even had we a satisfactory characterisation of the masked cases, a second lethal consequence looms large. In order to preserve seamlessness, we now need to avoid the postulation of a sharp boundary between a last Φ and a first exemplar of this new genre of indeterminate cases, the non- tacitly polar instances of the Basic Formula applied to Φ and Ψ (let's call these the Δ's.) So, on the Buffering View, we now need in turn to buffer the contrast between Φ and Δ, however exactly the instances of the latter are to be described. But strategically, the means at our disposal are just the same as— and hence no better than—those just deployed for the Φ–Ψ distinction, — except that now, of course, there are fewer elements to subserve the buffering of the distinction, since the Φ–Δ series *is shorter than* the Φ–Ψ one. Since exactly the same form of problem is going to recur at every stage and the series is finite overall, the strategy cannot succeed.

The root of the trouble is that there is, simply, no satisfactory conception of what a borderline case is that is serviceable for the explanation of seamlessness. Obviously no "third possibility" conception is to the purpose: if one is trying to explain seamless transition between contrasting situations, it doesn't help to interpose a third category of contrasting siltation. But if, recoiling from that, we essay to think of the interposed category as compatible with each of the originally contrasted statuses (so dropping Dummett's principle), then in assigning an object to that category we fall silent concerning what if any shift from polar status it instantiates. To fall silent, is not to explain anything. Moreover, when pressed, as we saw, it seems we are forced to postulate a "Third Possibility" type of case— Δ-cases — after all. At which point, the game is effectively lost.

We should conclude that there is no prospect of a stable elucidation of seamless transition by means of the conception of an endless hierarchy of orders of borderline cases. So far from being well motivated by the possibility of seamless transition between instances of incompatible vague predicates, the Buffering View winds up in compromise and confusion.

Where does that leave the Transition problem? Well, it is striking that the kind of difficulty just outlined will afflict *any* attempt to do justice to the nature of the changes, stage by stage, involved in a process of seamless transition across a finite series of stages between contrary poles. It has nothing especially to do with vagueness or our having recourse to the notion of a borderline cases. For suppose we have somehow turned the trick: we have somehow succeeded in fully correctly describing, stage by stage, a process of seamless transition. We will have had to say incompatible things about some of the stages. Let m and n be a pair where we did that and which are as close together as any pair where we did that. They will not have been adjacent. Let F be the description given of m, and G that given of n. So m′ will have received a verdict, F′, compatible with both F and G. Is F true of m′? If it is, then G isn't. So, since compatible with G, F′ doesn't do full justice, in relevant respects, to m′, even if true of it. So if we did somehow do full justice to all the stages, F cannot be true of m′. But then the series wasn't seamless after all: there is a sharp boundary at m.

Conclusion: the Transition problem is insoluble in *any* vocabulary if the 'full justice' requirement is enforced. So far from demanding recourse to an apparatus to describe borderline cases of arbitrarily high orders, the requirement that seamless transition somehow allow of a fully adequate description, stage by stage, is unsustainable. If explaining how seamless transition is possible involves doing justice, in all relevant respects, to the elements in a finite series that manifests as effecting such a transition, it is about as helpful to believe in higher-order vagueness as in fairies.

Dissatisfaction may persist. Forget about doing full justice to seamless transition. Don't we at least have invoke concepts of higher order vagueness and buffering if we are to describe the relevant kind of series in a fashion *consistent* with seamless transition, even if the description does not do full justice to it? Well, no. Once the 'full justice' requirement is relaxed, and we need merely to avoid adjacent incompatibilities, we can perfectly well describe the stages of a seamless transition, without misrepresentation using only *precise* vocabulary. Thus suppose Johnny grows seamlessly from 5 feet

tall to 6 feet tall between his fourteenth and eighteenth birthdays and consider the series of appropriately dated true (let's suppose) descriptions:

Johnny is exactly 5 feet tall
Johnny is exactly 5 feet tall, give or take an inch
Johnny is exactly 5 feet 1 inch tall
Johnny is exactly 5 feet 1 inch tall, give or take an inch

…and so on.

One last try. Notice that when the admissible substitutions for 'F' are restricted to predicates in the range used in the description of Johnny's changing height, the result is not, of course, to provide a model of the original no-sharp boundaries principle,

(i) $\sim(\exists x)(Fx \ \& \ \sim Fx')$

— for any choice of F in the range concerned, there will be a last case of which it is true. By contrast, isn't it forced on us that each of the hierarchy of Fara's Gap principles *is* true in a finite series exemplifying seamless transition between instances of contrary vague concepts? If so, then at least from a classical point of view, that enforces *acceptanc*e of the hierarchy of borderline kinds, even if we are thereby no better placed when it comes to doing justice to the phenomenon of seamless transition.

But this has to be a bad thought. If, after we introduce the Definiteness operator, seamlessness enforces the Fara Gap principles, then before we introduce the Definiteness operator, it enforces the major premise of the No-Sharp-Boundaries paradox. What we considered earlier was an argument, impressive in the context of classical logic, that (i) is not an adequate capture of F's vagueness, which is rather canonically expressed by

(ii) $\sim(\exists x)(\mathit{Def}Fx \ \& \ \mathit{Def}\sim Fx')$.

Let that conclusion stand. Then the vagueness of F, qua canonically expressed by (ii), does not impose (1). But nothing has been done to disarm the impression that the seamlessness of the relevant transition does. That is another matter. If seamless enforces the higher-order Gap principles, it enforces (i) too, and the No-Sharp boundaries paradox re-arises as a paradox of seamlessness. There are two directions on which to look for a response to the situation. One, proposed recently by Fine,[28] is to restrict the underlying logic of negation in such a way a to block the 'right-to-left'

[28] In his monograph [in progress]. Fine rejects the rule of Conjunctive Syllogism:

$$\frac{A, \sim(A\&B)}{}$$

reasoning of the No-Sharp-Boundaries paradox. In that case, (i) and the members of the hierarchy of Gap principles will all be acceptable as mandated by seamlessness, however inchoately understood. My own preference though, is to explore the thought that relevant instances of 'unpalatable existential' claims of the form,

$$(\exists x)(Fx \ \& \sim Fx'),$$

are rendered ungrounded, rather than false, by phenomena of seamless transition, which therefore requires less inchoate understanding, and that F's vagueness in the relevant series likewise renders the unpalatable existential ungrounded. I have no space here to pursue these suggestions.[29] In any case, enough has been done, I trust, to disarm the Seamlessness intuition as a motive for the Buffering view.

VIII

The Ineradicability intuition again

It remains to re-scrutinise the Ineradicability intuition, expressed in rather different ways by Dummett and Russell. Both implicitly start from the idea of the vagueness of the borderline between Φ and Ψ as consisting in a region of uncertainty — a 'penumbra' in Russell's seminal image — and envisage an additional stipulation to try to bring this region under linguistic control: a new predicate in Dummett's case, a moratorium on description in Russell's case (and a third move would be to extend Ψ, if it is the complement of Φ, — or in any case, to extend the sphere of application of one of the concepts concerned.) Both then simply assert — plausibly but, notably, without any argument whatever — that the proper application of the new stipulation will itself be vague: that there will be cases of which it is uncertain how to apply the new term, or whether they fall within the scope of the moratorium.

Now it should, on reflection, seem puzzling that this is so much as a plausible assertion. To claim that there are borderline cases of a certain concept is, after all, partly a sociological claim: it is

~B

[29] Wright [2001], [2004] and [2007] offer argument in some detail that acceptance of a predicate's vagueness need not involve denial of a relevant unpalatable existential, i.e. endorsement of an instance of (i). those arguments, if effective, equally militate against acceptance of higher-order Gap principles as a response to the vagueness of the predicates concerned. I have not elsewhere attempted to explain why seamlessness, properly understood, should not motivate acceptance of Gap principles. But the basic point is that seamlessness is an epiphenomenon of our discriminative limitations. It is merely a projective error to read it back into the characterisation of the elements in a seamless series.

to predict that possessors of the concept will not react with verdicts about its application that collectively configure a sharp distinction between positive and negative cases. How do Russell and Dummett know this? Who is to say that, after "eminence" was introduced in the manner Dummett envisages, we would not respond with a stable, consensual practice converging on an agreed range of applications for all three concepts—*hill*, *eminence* and *mountain*—and responding in no case with the characteristic manifestations of vagueness? So why is our reaction to the ineradicability claim not, "How do you know?" Why is the claim plausible at all? Why don't we try it out and see?

The answer, presumably, is that the empirico-sociological prediction is itself plausible— we think we know already what the outcome of an experiment would be. But how can that be? — It is not, after all, as if we have often made stipulations of the Dummett-Russell sort and experience has taught that they do not work. Rather, the explanation must have something to do with a sense of the limited nature of the guidance that the envisaged kind of stipulation would be able to provide. In going along with the prediction of ineradicable vagueness, we are reporting something about our own sense of limitation in response to the kind of stipulation envisaged; the phenomenon is broadly — not exactly — of a piece with one's ability to predict uncertainty in one's applications of rules which one knows one has only partially understood; or to reliably respond to the question, "Do you understand?", when what is at issue is competence for some form of subsequent task. I'll enlarge on that diagnosis in a moment. The immediate point is that we can hardly expect to be clear about whether or not the ineradicability intuition does indeed imply the Buffering view or anything else it may imply before we understand *why it strikes us as correct*.

There is an objection to the alleged connection between the ineradicability intuition and higher-order vagueness that is worth noting before we get into that issue. It is that a so far unremarked additional presupposition is required before any connection is even apparent. That presupposition is that the introduction of a linguistic stipulation of the kind envisaged by Russell and Dummett will have no impact on the identity of the concept —Φ— whose borderline cases it aims to provide means of denoting or otherwise differentially treating. This is actually quite implausible. Consider a small child tidying up his play-bricks, so far without any colour words save "red" "blue" "green" and "yellow", who is told to put the reds into one bin and the blues into another, although the bricks include many shades of red, blue, mauve, purple and pink. It seems quite expectable that he will place many reddish purples and bluish purples in the red and blue bins respectively which, if we were to single out a few royal purple bricks and others of similar shades, and give him the word and

a new bin with the instruction to tidy the purples into it, he would then prefer to house there. In general, it is to be expected that provision of the resources to mark an intermediate category will have the effect of disturbing — narrowing — the extensions of the concepts which flank it to include fewer uncomfortable cases— and thereby of modifying the original concepts themselves. But if the effect of regulating the response to the borderline cases would be to modify the concepts concerned, then the ineradicability intuition provides no argument for thinking of them as being even second-order vague—rather we have a situation where the introduction of the new resources afforded by the stipulation merely generates *three new* concepts which then exhibit ordinary — *first-order* — vagueness in relation to each other.

This is a good point. But I do not think it takes us to the heart of the issue. There is a *second* questionable assumption at work in Dummett's and Russell's line of thought — which indeed is still at work even in the objection just expressed. It is the assumption that that the invitation to annex a new word to the borderline cases of a distinction, or to respond to them with a moratorium on classification, or some other kind of new, distinctive treatment, is one that *can be taken up*. In order to respond to such an invitation, one must first be able to corral the borderline cases— those, after all, are the only cases to which the new practice, whatever it involves, is to be applied. The question this goes past is whether the reactions that characteristically manifest the borderline status of a case involve the exercise of a concept somehow contrasting with the polar concepts; or whether what they betray is, rather, a subject's difficulty in bringing it under one of the polar concepts— a "drying of the springs of opinion". If it is the latter, then the reason why the invitation will not have the effect of generating precision—a new, sharply tripartite practice of some kind—is not because the separation between the cases to which the new convention is to apply and the rest is itself vague on both borders, but because we have no settled concept of those cases in the first place.

We need to go carefully here. It would be unmotivated to deny, of course, that there is such a thing as the judgement that a case is borderline. The question concerns what is the *content* of this judgement: in particular, whether it involves bringing the case in question under a concept contrasted with the polar concepts, — so that it's force, like theirs, is *normative* (say: "Here you should not take a view — the case is too far removed from the clear cases") — or whether it is, rather, a projection of the characteristic phenomenology of attempted judgement in the particular case, so that its force is broadly sociological (say: "Here people have weak and unstable views, struggle to come to a view, etc."). The difference is crucial. The roots of the Buffering view of higher-order vagueness lie

entirely in the former way of thinking. The reason why we do not eradicate vagueness by proposing a new form of classification of the borderline cases is not because the concept — say*: case too far removed from the clear cases* —that would govern it is itself vague —but because it would be governed by *no* concept. We don't have a conception of the kind, *borderline case of the* Φ - Ψ *distinction*, to regulate the new practice by.

This is the point of connection, bruited above, with the phenomenon of avowably imperfect understanding. The reason why it may be confidently predicted that a Russell/Dummett stipulation will not have the effect of introducing precision is indeed comparable the reason why I may confidently predict that I will not be able to give the right answers in future applications of a rule whose explanation I realise I have imperfectly followed. (Of course, in both cases there is the bare possibility that we will surprise ourselves.) Simply: I do not know how to apply such a stipulation because I lack any stable concept of the kind of cases to which it is intended to apply — my characteristic reaction to such cases is precisely not to be able to form a stable view about whether they are cases of the sort to which the new stipulation is *not* to apply: the Φs and the Ψs. Again: if one's characteristic reaction in the borderline area is a drying of the springs of opinion,—an inability to bring a case under either polar concept and associated uncertainty whether it should be so brought, — then of course the invitation to, say, introduce a new predicate, covering cases whose status is to *contrast* with polar cases, will not result in precision—in confident and complete classifications across the range. The content of the quandary was precisely whether to apply a polar concept; so the invited new predicate, the application of which will pre-empt the polar concepts, will be bound to inherit some of that uncertainty.

There is, of course, what we might term the *sociological option*: to annex a sociological conception of the borderline cases of a distinction to a stipulation of the Russell/Dummett kind. (In the case of a single judge, 'borderline case' will then become a concept grounded in his own characteristic psychological reactions.) But the obvious point to make is that no such conception of the borderline cases of Φ gives any literal sense to the idea of the boundary between the Φs and the borderline Φs being vague. As a first approximation: if the content of a judgement that a case is borderline is broadly sociological, or psychological, then whereas in judging that a case is Φ, we are making a judgement about the case, in judging that a case is borderline Φ, we are making a judgement about *us*; so the idea that this distinction might itself be vague makes no sense — mutual vagueness requires a common domain of predication.

One way of characterising the root error in the Buffering view is that it depends on thinking of the borderline cases of a vague distinction as if they were shades of purple and the given distinction were like that between red and blue. Obviously that cannot be the nature of the general run of cases. In particular, it cannot be the nature of the distinction between the Φs and the non-Φs. And even putting that case to one side, there is an intuitive notion of *adjacency* for vague concepts that compete in a single space — whereby red and orange, for example, or blue and purple are adjacent in colour space, or *moderately uncomfortable* and *painful*, perhaps, are adjacent in the space of sensations. Where concepts are adjacent, we will have no third concept to characterise a buffer zone between them, in the way in which purple buffers the blues and reds. We may indeed be able to retain a narrower concept that applies in the borderline area (for example, *blood orange*), but this will not compete with the originals (*red* and *orange*) as they compete with each other. And if we annex a word to it, the result will be the narrowing phenomenon noted above.

The root error in the Buffering view is to think of borderline cases as instances of what I have elsewhere called Third Possibility. I have given other arguments against that broad conception and will not rehearse them here.[30] The Ineradicability intuition is indeed a commitment to the Buffering view when taken under the aegis of Third Possibility. And the lesson to learn is that the inference of buffering from ineradicability goes wrong by—draws the wrong conclusion as a result of—passing over the conception of mutually vague concepts not as demarcated from their neighbours by a borderline area conceived on Third Possibility lines but as adjacent: as running right up against each other in a way that, in contrast with the situation of precise concepts, confounds our powers of conceptualisation. GLOSS That, in brief, is the cardinal source of the illusion of second-order blurred boundaries. No doubt much more needs to be said.[31]

Department of Philosophy, New York University
Arché Research Centre, University of St. Andrews

---

[30] For elaboration, see Wright [2001], [2004], and [forthcoming].

[31] I am grateful to the members of Arché's AHRC-funded project on *Vagueness: its Nature and Logic* (2003-6) for helpful discussion and critical comments during the seminars that saw the gestation of this paper. My special thanks to Elia Zardini, who gave me detailed written comments on the draft I prepared for the 2007 Arché conference, and to Mark Sainsbury, my commentator on that occasion. A proper response to all their observations and suggestions would have required a much more extended and doubtless much improved treatment.

References

Beall, JC. Ed., [2004] *Liars and Heaps: New Essays on the Semantics of Paradox* (Oxford: Oxford University Press)

De Clercq, R. and Horsten, L. [2004] "Perceptual Indiscriminability: In Defence of Wright's Proof", *The Philosophical Quarterly* 54, 439-44

Dorr, C. [2009] "Iterated Determinacy", this volume, XXX-XX

Dummett, M. [1959] "Wittgenstein's philosophy of Mathematics", *The Philosophical Review*, LXVIII, 324-348; reprinted in Dummett [1978] at 166-85

[1975] "Wang's Paradox", Synthese 30, 301-24; reprinted in Dummett [1978] at 248-68

[1978] *Truth and Other Enigmas* (London: Duckworth)

Fara, D. [2001] "Phenomenal Continua and the Sorites", *Mind* 110, 905–935. Published under the name "Delia Graff".

[2004] "Gap Principles, Penumbral Consequence, and Infinitely Higher-Order Vagueness" in Beall, JC, ed., [2004], 195-221, Published under the name "Delia Graff"

Fine, K. [1975] "Vagueness, truth and logic", *Synthese* 30, 265-300

Hyde, D. [1994] "Why Higher-Order Vagueness is a Pseudo-Problem", *Mind* 103, 35-41

[2003] "Higher-Orders of Vagueness reinstated", *Mind* 112, 301-5

Keefe, R. and Smith, P., Eds., [1993] *Vagueness; a Reader* (Cambridge, Mass.: Bradford MIT Press)

McGee, V. and McLaughlin, B. [1995] "Distinctions Without a Difference", *The Southern Journal of Philosophy*, supplementary volume XXXIII, 203-51

Russell, B. [1923] "Vagueness", *The Australasian Journal of Psychology and Philosophy,* 1, 84-92; reprinted in Keefe, R. and Smith, P., eds., [1996], 61-68

Sainsbury, M. 1990] "Concepts without boundaries" (London: King's College, London - Inaugural lecture) Reprinted in Keefe and Smith (eds.) [1996], 251–264.

[1991] "Is there higher-order vagueness?" *Philosophical Quarterly*, 41, 167–182

[1992] "Sorites paradoxes and the Transition Question" *Philosophical Papers* 2, 177–189

Soames, S. [2003] *Understanding Truth* (New York: Oxford University Press)

Sorensen, R. [1985] "An Argument for the Vagueness of "Vague" ", *Analysis* 27, 134-37

Varzi, A. [2003] "Higher-Order Vagueness and the Vagueness of "Vague" ", *Mind* 112, 295-99

Williamson, T. [1990] *Identity and Discrimination* (Oxford: Basic Blackwell)

[1994] *Vagueness* (London: Routledge)

[1997] "Imagination, Stipulation and Vagueness", *Philosophical Issues*, 8, Truth, 215-228

[1997a] "Reply to Commentators: [Horgan, Gomez-Torrente, Tye]", *Philosophical Issues*, 8, Truth, 255-265

[1999] "On the Structure of Higher-Order Vagueness", *Mind* 108, 127-43

Wright, C. [1975] "On the Coherence of Vague Predicates", ", *Synthese* 30, 325-65

[1987] "Further Reflections on the Sorites Paradox", *Philosophical Topics*, XV, 227-290

[1992] "Is Higher Order Vagueness Coherent" *Analysis* 52, 129-39

[2001] "On Being in a Quandary: Relativism, Vagueness, Logical Revisionism" *Mind* 110, 45-98

[2003] "Vagueness: a Fifth Column Approach" in J.C Beall, Ed., [2004], 84-105

[2007] " 'Wang's Paradox' ", in *The Philosophy of Michael Dummett*, The Library of Living Philosophers volume XXXI, edited by Auxier, R and Hahn, L (Chicago: Open Court), 415-44

[forthcoming] "On the Characterisation of Borderline Cases" in *Meanings and Other Things: Essays on Stephen Schiffer*, edited by Ostertag, G (Cambridge, Mass.: MIT press)