

Focus restored: Comments on John MacFarlane

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Accepted: 31 August 2007 / Published online: 30 October 2007
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Abstract In “Double Vision Two Questions about the Neo-Fregean Programme”, John MacFarlane’s raises two main questions: (1) Why is it so important to neo-Fregeans to treat expressions of the form ‘the number of Fs’ as a species of singular term? What would be lost, if anything, if they were analysed instead as a type of quantifier-phrase, as on Russell’s Theory of Definite Descriptions? and (2) Granting—at least for the sake of argument—that Hume’s Principle may be used as a means of implicitly defining the number operator, what advantage, if any, does adopting this course possess over a direct stipulation of the Dedekind-Peano axioms? This paper attempts to answer them. In response to the first, we spell out the links between the recognition of numerical terms as vehicles of singular reference and the conception of numbers as possible objects of singular, or object-directed, thought, and the role of the acknowledgement of numbers as objects in the neo-Fregean attempt to justify the basic laws of arithmetic. In response to the second, we argue that the crucial issue concerns the capacity of either stipulation—of Hume’s Principle, or of the Dedekind-Peano axioms—to found *knowledge* of the principles involved, and that in this regard there are crucial differences which explain why the former stipulation can, but the latter cannot, play the required foundational role.

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Keywords Neo-Fregean · Abstraction · Logicism · Singular reference · Implicit definition · Stipulation

Anything worth regarding as logicism about number theory holds that its fundamental laws—in effect, the Dedekind-Peano axioms—may be known on the basis of logic and definitions alone. For Frege, the logic in question was that of the *Begriffsschrift*—effectively, full impredicative second order logic—together with the resources for dealing with the putatively “logical objects” provided by Basic Law V of *Grundgesetze*. With this machinery in place, and with the course-of-values operator governed by Basic Law V counting as logical, it is possible for all the definitions involved in the logicist reconstruction of arithmetic and analysis to be fully explicit, abbreviative definitions. Had Frege’s project succeeded, he would therefore have been in position—by his own lights—to regard the axioms of number theory simply as definitional abbreviations of certain theorems of his pure logic.

Basic Law V, as every interested party knows, is inconsistent. But twentieth century orthodoxy would have scorned its description as a law of logic in any case, purely on the grounds of its existential fecundity. Contemporary Neo-Fregeanism in the foundations of mathematics does not, in intention at least, pick any quarrel with the idea that pure logic should be ontologically austere. It does however maintain that the existence of the natural numbers and the real numbers as classically conceived, and thereby the truth of the traditional axioms of arithmetic and analysis, may still be known a priori on the basis of logic and definitions. For the purposes of this claim, logic is once again conceived as essentially the system of *Begriffsschrift*. But Basic Law V is superseded by a variety of *abstraction principles*, of which Hume’s Principle is the best known example, which we are regarded as free to lay down as true by way of determination of the meaning of the non-logical vocabulary that they contain. Thus—the idea is—the Dedekind-Peano axioms, for example, may be known, a priori, to be true by virtue of their derivation in pure logic from a principle which may be regarded as stipulatively true, and whose very stipulation may be regarded as conferring content upon the sole item of non-logical vocabulary—the cardinality operator—which it contains and thereby as conferring content upon Hume’s Principle itself.

An epistemology of implicit definition is presupposed in this. It is presupposed that, in the best cases, it is possible—without any collateral epistemic work—stipulatively to associate a certain type of sentence containing previously undefined vocabulary with certain conditions of truth in such a way that the undefined terms take on meaning; and this in turn, moreover, in such a way that a recipient of the definition who thereby acquires a grasp of the meanings so conferred and comes to a belief in the truth of the sentence expressing the stipulation—the vehicle—comes to a *knowledgeable* belief. On this view of the matter, Hume’s Principle is viewed as a compendious stipulation associating the truth of each instance of its left hand side—an identity statement configuring the hitherto undefined cardinality operator—with the satisfaction of the corresponding condition of the right hand side, a statement of the existence of a one-one correspondence between concepts which, in basic cases, will presuppose no understanding of the cardinality operator. More generally, the idea is that by laying it down that Hume’s Principle is to hold, we may succeed in so fixing the meaning of the

cardinality operator that the resulting belief in the proposition expressed as a function of the new meaning ranks as knowledge.

There is, of course, a very great deal to say about this proposal. Some of it concerns the detailed working of the model demanded of (one kind of) basic a priori knowledge—what we have elsewhere called the “traditional connection”¹ between implicit definition and the a priori—and some of it concerns specific issues about whether abstraction principles—or more particularly, second order, impredicative abstraction principles like Hume’s Principle—can deliver what the traditional connection promises: can rank as good implicit definitions in the required sense. Well-known doubts about the latter explored in the literature include the Julius Caesar problem,² a range of problems connected with impredicativity itself,³ and of course the problem of “Bad Company”⁴ that provides the focus of the present volume: the problem of providing a principled account of the distinction between those—“good”—abstraction principles, like Hume’s Principle, or Cut Abstraction,⁵ for which the neo-Fregean wants to make the epistemological claims outlined, and a residue of formally similar principles—including Basic Law V, Boolos’ Parity Principle and its near relative, the Nuisance Principle, and the family of principles mischievously named Distraction Principles by Alan Weir⁶—about which those claims are unsustainable. There is a straightforward overarching connection, of course, between the issues concerning implicit definition and the Bad Company problem since—or so our working hypothesis has been—the solution to the latter has exactly to consist in a characterisation of which the good implicit definitions are and a demonstration that the intuitively Bad abstraction principles are exactly those that violate the various conditions of goodness in implicit definition. The final solution to the Bad Company problem awaits a finally adequate account of what conditions a principle must meet if its stipulation, or ungrounded acceptance as true, is to serve the conferral of meaning upon its primitive expressions in such a way that one who so stipulates or accepts it can know it (a priori) to be true without collateral epistemic work or hostage.

It is, of course, possible to take the view that there are no such conditions—that the conception of implicit definition that we are gesturing at, with its traditional connection to basic a priori knowledge, is illusory.⁷ But John MacFarlane gives powerful formulation to a problem about this aspect of neo-Fregeanism which has been

¹ Hale and Wright (2000, p. 117).

² For Frege’s statement of the problem, see Frege (1884), Sects. 55–6, 66–7. Subsequent discussion includes Wright (1983, pp. 107–117), Dummett (1991, Chaps. 13, 15, 17), Hale (1994, Sect. 3), Sullivan and Potter (1997), Hale and Wright (2001b), Stirton (2003).

³ Published discussion includes Dummett (1967), Wright (1983, pp. 139–145, 180–184), Dummett (1991, Chap. 18), Hale (1994, Sect. 6), Wright (1998a, b), Dummett (1998).

⁴ The term was originally employed in this context in Wright (1997, p. 212).

⁵ As used in Hale (2000a). Cook (2001) criticizes certain generalisations of Cut Abstraction, but Hale (2000b)—without extraordinary prescience; the actual publication date was 2002!—argues that, in so far as these may be objectionable, an abstractionist need not be committed to them.

⁶ For Boolos’s Parity Principle, see Boolos (1990, pp. 214–215), for Nuisances, Wright (1997, Sect. VI), for Distractions, Weir (2003).

⁷ Sceptics include Paul Horwich—see Horwich (1997, 1998)—and Timothy Williamson—see, for example, Williamson (2003, 2006).

in the wind for a while, and which is importantly independent of that scepticism. MacFarlane, at least for the purposes of his present discussion, raises no objection to the “traditional connection”, nor to the contention that Hume’s Principle, in particular, is one of the good cases of it. His question is: *If* Hume’s Principle is a good case, delivering all that the epistemological benefits the neo-Fregean claims for it, what—in view of the equivalence of *Frege arithmetic* with regular second order *Peano arithmetic*⁸—would be lost if instead of stipulating Finite Hume’s Principle (henceforward simply *Hume*) and then deriving the Dedekind-Peano axioms (henceforward simply *Dedekind-Peano*) as theorems (i.e., proving ‘Frege’s Theorem’), we were simply to stipulate Dedekind-Peano outright? What does the neo-Fregean have to say to the suggestion that the epistemology of implicit definition that he painstakingly prepares to authenticate his use of abstraction principles could as well be deployed directly to authenticate the axioms which are his ultimate goal? In short, if neo-Fregeanism can be made to work in the setting provided by the kind of account of implicit definition that we aim to give, why doesn’t *neo-Hilbertianism*?

This is, in our estimation, by far the more important of MacFarlane’s “Two Questions” and it will occupy the bulk of our reply. But his other question, concerning the neo-Fregeans’ treatment of their canonical terms for numbers as species of singular term, also raises a point of interest and deserves a response. What would be lost, he enquires, if we didn’t take those terms as singular terms at all, but accepted something like Russell’s own account of them as quantifier-phrases of a certain kind? We will begin by saying, briefly, what would be lost.

1

The basic neo-Fregean claim about Hume’s Principle, then, is that its stipulation serves implicitly to define the cardinality operator, ‘the number of ...’, thereby simultaneously providing for the introduction of a range of complex singular terms formed by filling the argument-place by a suitable general term or concept-word, and—though this needs additional argument—explaining a sortal concept of (cardinal) number, under which fall any objects singled out by terms of that type. In MacFarlane’s view, there are grounds to question whether numerical terms of the form ‘the number of Fs’ are properly viewed as singular terms—as purportedly presenting a range of objects—at all, and significant costs, in the form of complications of logic, consequent upon so treating them. Notably, he makes no claim that either of these considerations is decisive, but presents them primarily as a means of reinforcing the challenge to explain why, if it is, it is essential to our project to treat numerical terms as singular, and what, if anything, would be lost if we were to adopt instead a version of Hume’s Principle

⁸ *Frege arithmetic* is the system of consisting of Hume and full impredicative second order logic, and regular second order *Peano arithmetic* that consisting of the standard Dedekind-Peano axioms, including the second order induction axiom, with full impredicative second order logic. The two systems are each categorical, with the same standard models. They are also equivalent in the stronger sense that subject to natural bridge principles (essentially Frege’s definitions of the primitive vocabulary of arithmetic), Finite Hume’s Principle and the standard Dedekind-Peano axioms are also proof-theoretically equivalent (in the setting of full impredicative second order logic), see Heck (1997).

featuring definite descriptions construed as quantifiers. So it would be inappropriate to discuss these ‘softening up’ moves at any great length. But some brief comments on them will assist our response to his main question.

We are, first, less than impressed by the syntactic/inferential similarities to which MacFarlane seems inclined to attach considerable weight, between definite descriptions and explicit natural language quantifier-phrases, centred on the principle:

Conservativeness: $[\text{Det } x: Fx]Gx \leftrightarrow [\text{Det } x: Fx](Fx \wedge Gx)$

MacFarlane points out that definite descriptions conform to this principle along with undoubted quantifier phrases such as ‘A woman’, ‘No American drivers’ and ‘Most goldfish’. Granted—but how significant is the similarity? After all, *complex demonstratives* such as ‘this book’, ‘that bottle’, etc., likewise obey the principle, so that we have e.g.

This book is boring \leftrightarrow this book is a book that is boring

Does that give us a good reason to group *them* with quantifier phrases and deny that they are singular terms?⁹

Second, even if this or other evidence weighed in favour of treating many definite descriptions—identified much as Russell originally proposed¹⁰—as disguised quantifiers, there might be good grounds for refusing to extend this treatment willy-nilly to all members of the class. In particular, it is not clear that it should be extended to what one might call functional terms, such as ‘the direction of the line connecting Aberdeen and Birmingham’, in contrast with terms like ‘the square of 17’. One may plausibly regard the latter as equivalent to, and perhaps as analysable as, the definite description ‘the number which results from multiplying 17 by itself’. But the plausibility of so regarding it depends upon there being a sortal concept of number in prior good standing. It is precisely because it isn’t clear that there is a relevant concept of direction which does not require taking its instances to be essentially ‘of’ lines that there is no natural or plausible analysis of the former as ‘the direction which is of the line connecting Aberdeen and Birmingham’. In such cases, it is least arguable that the use of the functional singular term is semantically and explanatorily prior to that of the corresponding general (sortal) term (‘direction’, etc.). More generally, it is arguable that in so far as the standard natural language paraphrase of applications of the cardinality operator to concept words represents them as semantic definite descriptions, it encourages, or betrays a definite mistake. Semantically, the apposite use of a definite description involves the satisfaction of a uniqueness condition: there has to be a unique object meeting the condition that the description operator binds. In order to substantiate such a constraint for the case of functional terms introduced by abstraction, one

⁹ We are well aware that some—see, for example, King (1999, 2001)—have been happy to embrace the conclusion that such demonstratives are indeed disguised quantifier-phrases. But one might, with at least as much justice and plausibility, regard the fact that the proposed test of quantifier-status pushes us into that conclusion as, rather, a *reductio* of the test.

¹⁰ That is, a definite description is any expression of the form ‘the so-and-so’ (or at least any such expression where the definite article is ‘strictly used’, so as to imply uniqueness—cf. Russell (1905, p. 44)). There is no denying that terms of the form ‘the number of Fs’ are definite descriptions in this sense, but that leaves entirely open the questions about their semantic architecture that we are about to raise.

therefore needs to associate the relevant functor—‘ Σ ’, say—with an underlying *relation* and then to think of ‘ $\Sigma(a)$ ’ as purporting to denote the unique object so related to a . Uniqueness fails just when there is more than one such object so related to a . This point is of the essence of the semantic composition of definite descriptions. So we need to ask: is there in general any conception of such a relation somehow conveyed as part of the sense attached to an abstraction operator by its implicit definition via the relevant abstraction principle? In the case of Hume’s Principle and the associated cardinality operator, glossed as “the number of”, the question becomes to identify an associated relation such that the sense of “the number of Fs” is to be conceived as grasped *compositionally*, via grasping this relation plus the presumption of uniqueness incorporated in the article. Uniqueness will be the effect of the many- or one-oneness of this relation—something that might ideally admit of proof. It is very doubtful however whether it is right to view the sense assigned to the cardinality operator by Hume’s Principle as compositional in this particular way.¹¹ And if not—if the operator is best conceived as semantically atomic—there is no case, or at least none made by MacFarlane, for treating the terms it enables us to form as semantically definite descriptive, or in any other way as other than singular-referential.

But thirdly, and much more importantly, the primary issue for the neo-Fregean is in any case not whether expressions of the form ‘the number of Fs’ are, as they are employed by English speakers, best regarded as a kind of singular term, or rather as a kind of quantifier-phrase. It is rather whether it is possible to *introduce* a class of cardinal-numerical expressions to serve as canonical devices of singular reference. That is, we are not concerned with a question of descriptive syntax and semantics, but with one concerning whether numerical expressions equipped to function as singular terms could be introduced by means of Hume’s Principle, and the corresponding sortal concept on number defined by ‘ x is a number iff for some F , x is the number of Fs’.

Yet why should *that* issue be what is of concern to us? This takes us to the principal question in this part of MacFarlane’s paper. Why should it matter to us whether the terms we seek to introduce by Hume’s Principle are properly viewed as semantically singular, rather than a species of quantifier-phrase, say?

There is a relatively simple and straightforward explanation. It seems to us, and we took it pretty well for granted, that people can and do engage in genuine *singular thought* about numbers, and that it ought, therefore, to be possible to introduce a range of terms to serve as the primary vehicles for the expression of such thought. The adoption of a ‘numerical quantifier’ version of Hume’s principle, as MacFarlane suggests,

¹¹ MacFarlane’s idea is that terms having the surface form ‘the number of Fs’ are constructed using the underlying relational expression ‘ x numbers the Fs’—but how is that expression understood? One can of course define it to mean ‘ $x = N_y:Fy$ ’—but this relational expression is evidently compositionally *posterior* to the cardinality operator. The question, for the viability of MacFarlane’s proposal, must therefore be whether ‘ x numbers the Fs’ can be defined independently, without presupposing prior understanding of numerical terms. It is certainly not obvious that it can be. But even if it can be, the more important issue for present purposes is not whether one could introduce the cardinality operator on the basis of such an underlying relation, but whether one can, as we contend, intelligibly introduce it on no such basis, as semantically atomic—if so, there is simply no case for the assimilation of numerical terms so explained, to definite descriptions, and consequently no case—or at least none based on a doubt about the point in the case of definite descriptions—for disputing that they are genuine (i.e., object presenting) singular terms.

would—though not actually incompatible with viewing numbers as possible objects of genuinely singular thought—leave this to be explained.

It may be asked: what does this matter, as far as providing a logicist foundation for arithmetic goes? How does the possibility of singular reference to and singular thought about the objects of arithmetic have any significant role to play in *that* project? Might that not just as well be carried through, if it can be done at all, without introducing the fundamental terms in a way geared to the expression of singular thought? That's a perfectly good question, and we have never claimed that the answer must be negative. Certainly it can't be ruled out—at least not without much further argument—that there may be a way of securing broadly logicist foundations for arithmetic without putting any weight on the ideas of singular reference to numbers, conceived as a species of object.¹² But that conception *was* integral to Frege's version of the logicist project, and it has remained so in ours. In part, this simply reflects the conviction that if we can give a workable philosophical foundation for arithmetic which respects the surface syntax of ordinary arithmetical statements—in particular, the prominence of apparent singular reference to numbers effected by simple numerals and complex numerical terms—then we should. Given a broadly Fregean conception of objects, as referents of actual or possible singular terms, taking that surface syntax at face value means recognizing numbers as a kind of object. And of course the recognition of numbers as objects plays a crucial role in the execution of the programme. Most obviously, the proof—sketched by Frege in *Grundlagen* Sect. 82–3—that every finite cardinal number is succeeded by another, presupposes that the numbers are objects, lying within the range of the first-order quantifiers implicit in Hume's principle. A foundation along anything much like the lines just envisaged must at some stage provide for singular reference to numbers. Hume's Principle does so right from the start, in the most direct way possible.

As to the matter of the alleged complications forced on the underlying logic by our treatment of numerical terms as singular—specifically, the need for some form of 'free' logic—we can here be quite brief, in part because we seem to be in no very substantial disagreement with MacFarlane on the point, and in part because we have discussed the matter elsewhere.¹³ For those who like their logic classical in all respects, it will no doubt present as an advantage of treating numerical terms along Russellian definite-descriptive lines that doing so will permit retention of classical logic, and as a punishment for treating them as devices of singular reference that doing so requires—given that they may not be presumed non-empty—adopting a free logic. But it would be a mistake to attempt to make very much of this point, for at least two reasons. Firstly, the departures from classical logic needed to accommodate the fact that singular terms in general, and numerical terms in particular, may not be presumed to refer are, as McFarlane himself observes, quite modest. Since we take atomic senten-

¹² Prioritisation of numerically definite quantifiers—quantifiers of the ilk: *there are exactly n Fs*—is the central plank of David Bostock's neo-logicist project in *Logic and Arithmetic* (Bostock 1974, 1979). Michael Dummett is severely critical of Frege's own arguments against such an approach (see Dummett (1991, Ch. 9)). An early discussion of some of the material issues is Wright (1983, Sect. VI).

¹³ See our reply to Ian Rumfitt in Hale and Wright (2003, pp. 258–260), and Sect. 5 of Hale and Wright (forthcoming a).

tial contexts (including identity-contexts) to be true only if their ingredient singular terms refer, but must allow that at least some non-atomic contexts involving empty singular terms may be true, we must restrict the first-order universal instantiation and existential generalization rules. It would suffice¹⁴ to require, for universal instantiation, a supplementary premise asserting the existence of a referent for the instancial term, with a similar restriction on existential generalization. Second, the crucial proofs of the existence of numbers from Hume's principle are completely unaffected by the resultant free-logical setting—provided that identity-contexts are so understood that they cannot be true unless their terms have reference, the relevant proofs go through, whether the underlying logic is classical, or free in the above sense.¹⁵ In this sense, the issue of free logic is something of a red herring in this context—indeed, in view of what some have tried to make of it, a red whale.

¹⁴ Actually, the situation is more complex than usually recognised. We are allowing for the possibility of empty terms, whether simple or complex. Suppose we take it that, at least in the case of atomic sentences, reference failure always results in falsehood. Then it is possible for $(\forall x)A(x)$ to be true but an instance $A(t/x)$ to be false. Hence we must have a restriction on $\forall E$. However, if lack of reference for t does not invariably render a context $A(t)$ false, the usual restriction will actually be more stringent than needed. Call a context $A(t)$ *reference-demanding* with respect to t if $A(t)$ can't be true unless t refers. Then the *minimal* restriction on $\forall E$ will call for the supplementary premise only when $A(t/x)$ is reference-demanding.

This is relevant to the question of what restriction is needed on $\exists I$. If $A(t)$ is *always* false when t is empty, then there need be *no* restriction on $\exists I$ —since there will then be no case in which t is empty, $A(t)$ is true but $(\exists x)A(x/t)$ is false. But if one takes some contexts $A(t)$ to be non-reference-demanding, we may have $A(t)$ true but $(\exists x)A(x/t)$ false (because no object in the domain satisfies $A(x)$). In that case we must restrict $\exists I$ as well, by requiring the supplementary premise when $A(t)$ is non-reference-demanding.

A number of critics, including especially Shapiro and Weir (2000), and Rumfitt (2003) have suspected a can of worms for abstractionism around the issue of balancing the need for freedom in the underlying logic with its possession of sufficient strength to subserve proofs of the existence of the requisite abstracts. We see no problem. As noted, some of the points made in the text above are anticipated in our (2003) in response to Rumfitt. But an explicit and self-contained treatment of the issues is clearly desirable. We hope to offer this in future work.

¹⁵ This proviso is needed, of course, because the proof of the existence of $NxFx$ involves a step of existential generalization from $NxFx = NxFx$. So long as the identity is understood as incapable of truth unless its terms refer, no *additional* premise is needed for the proof to be equally good with the free logical rules suggested. (It is sometimes proposed that ' $t = t$ ' be understood so as to be true even if its ingredient term lacks reference. But we are under no pressure to adopt such a view.) It is worth remarking that the initial step in the proof, taking us from Hume's principle to: $NxFx = NxFx \leftrightarrow F$ is 1–1 correlated with F , proceeds in accordance with an *unmodified* second-order universal instantiation rule, with no requirement for a supplementary premise asserting the existence of a referent for ' F '. This is reasonable, provided we adopt what is sometimes called an 'abundant' conception of properties—that is, roughly, one according to which any well-formed predicate possessed of a sense is thereby guaranteed reference to a corresponding property. Further discussion is beyond the scope of this note. We have somewhat more to say on the matter in Hale and Wright (forthcoming a), and much more in Hale and Wright (forthcoming b). The prototype of the contrast between "abundant" and correspondingly "sparse" conceptions of properties is important for the purposes of understanding the neo-Fregean perspective on the existence of abstracta, and will occupy us again in the sequel.

2

In the discussion of ours¹⁶ to which MacFarlane is principally reacting, several constraints are proposed—in an acknowledgedly incomplete treatment—to distinguish good implicit definitions, capable of subserving the traditional connection, from bad. These include *Consistency*, *Conservativeness*¹⁷—which of course implies consistency—*Generality* and *Harmony*. It is striking that MacFarlane moves directly to formulate his challenge to us as, “Which of these four constraints do the Peano axioms fail to satisfy, and why?” If none, he continues, then it seems we will have to say either that both Hume and Dedekind-Peano are unsatisfactory as implicit definitions, or that both are satisfactory. In the first case, the neo-logicist treatment of arithmetic based on Hume is unsuccessful; in the second, it is unnecessary. But things are moving very quickly here. After all, our discussion also canvassed a constraint of avoidance of *Arrogance*—the situation where the truth of the vehicle of the stipulation is hostage to the obtaining of conditions of which it’s reasonable to demand an independent assurance, so that the stipulation cannot justifiably be made in a spirit of confidence, “for free”—and explicitly emphasised the centrality of this constraint in any account of implicit definition that is to subserve the traditional connection with a priori knowledge. MacFarlane is aware of this, of course, and cites passages from our writings in which we connect this requirement with the essentially conditional character of admissible implicit-definitional stipulation, citing the example of the two proposed stipulations:

J Jack the Ripper is the perpetrator of this series of killings, and

CJ If anyone singly perpetrated these killings, it was Jack the Ripper,

each presented as an implicit definition of the name, “Jack the Ripper”. The former arrogantly presupposes that there was a unique perpetrator of the killings; the latter, by contrast,—although its semantic purport for “Jack the Ripper” is essentially the same—avoids that presupposition by its resort to the conditional form. The distinction is clearly crucial if the traditional connection is to be saved, since of the two stipulations only CJ has any plausible claim to express an a priori knowable truth. But Macfarlane is less impressed by it than we are. For one thing, as he observes, Conservativeness would already exclude J, since it implies something new about the old ontology, namely that no more than one assassin was involved in the killings in question. So this example and its ilk provide no clear motive for an additional constraint. For another, it is in any case merely the stroke of a pen to cast any implicit definition in the desired conditional form. Rather than stipulate the conjunction of the Dedekind-Peano axioms, for example, we can instead stipulate a biconditional of which their conjunction comprises one constituent while a logical truth supplies

¹⁶ Hale and Wright (2000).

¹⁷ In a sense akin to that of Field (1980, pp. 8–12) whereby a definition (or theory) is conservative with respect to a theory T just in case its adjunction to T implies no new theorems about the ontology of T. There are, of course, issues about how best to formulate this constraint exactly. One formulation is provided at Wright (1997, p. 297). For detailed discussion, see Weir (2003, Sect. 3).

the other. No doubt, MacFarlane allows, such a stipulation is conditional only in a “Pickwickian sense” but—

... this is hardly an objection that a neo-logicist can make! [Hume’s principle] too makes the existence of numbers conditional on logical truths: that is precisely why it can serve as the basis of a kind of logicism.¹⁸

We’ll come back to these thoughts in a moment. Even if MacFarlane were right that an anti-arrogance, or Conditionality, constraint is ill-conceived, it is worth briefly reviewing his grounds for thinking that the respective stipulations of Hume and of the Dedekind-Peano axioms are otherwise on an equal footing, as far as the other four constraints are concerned. Clearly—in view of the equivalence of the systems of Frege arithmetic and Peano arithmetic¹⁹—Hume is conservative (and consistent) if and only if Dedekind-Peano are, so no differentiation is to be made on that score. But the situation with Harmony and Generality is a little less clear cut. Generality in the relevant sense—the sense of Gareth Evans’ well-known *Generality Constraint*²⁰—is the requirement, hard to characterise precisely, that an expression has been properly endowed with meaning only when made capable of figuring significantly in every type of context appropriate to its syntactic category.²¹ One of the concerns raised by the Julius Caesar problem concerns exactly this point: it is not implausible to think that the sense of a range of terms has been properly explained only when the relation—coincidence or otherwise—has been explained between their purported referents and items falling under antecedently understood sortal concepts and categories. If, as we have argued elsewhere²²—of course, the claim is controversial—Hume’s Principle itself contains resources to address this issue for the case of the numerical terms it serves to introduce, it appears by contrast that nothing is accomplished in this regard by a stipulation of Dedekind-Peano for “0” and the various terms formed by iteration of the successor functor. As structuralists never tire of pointing out, any progression—omega-sequence—of elements serves as well as any other as the domain for a model of Dedekind-Peano. In stipulating merely that those axioms are true, we have done nothing to constrain the identification of their referents beyond the requirement that they be capable of forming an omega-sequence. On the matter of Generality, then, MacFarlane’s question—which of these constraints do Dedekind-Peano fail?—is not to be supposed rhetorical. The issue is one aspect of the question of the meaning-conferring potential of the two proposed stipulations; we shall return to this more generally in the next section.

What of Harmony? Understood—as we intended—as a generalisation of the virtuous relationship in which introduction- and elimination-rules of deduction for a logical operator stand when the strongest consequences elicitable by an application of the elimination rule are exactly—no more, no less— what are independently assured by the premises for the introduction rule, it is a triviality that Hume’s principle,

¹⁸ The quotation is from the penultimate page of MacFarlane’s paper in this volume.

¹⁹ In the standard second-order logical setting—this qualification will be important below.

²⁰ See Evans (1982, pp. 100–105).

²¹ For some discussion, see Hale and Wright (2001a, pp. 134–135, 341–345).

²² The argument has several incarnations—see Wright (1983, pp. 107–117); Hale (1994, Sect. 3); Hale and Wright (2001b).

conceived in the natural way as such a pair of schematic rules, is harmonious.²³ Whereas the constraint might appear simply to have no application to a stipulation of Dedekind-Peano. MacFarlane is fully sensible of this, of course, and accommodates the point by suggesting a more flexible characterisation of the constraint:

If an expression is introduced by means of multiple implicit definitions, they must work together in a way that makes sense: for example, elimination rules should not be weaker than is justified by the introduction rules,²⁴

then observing that

... the Peano axioms work very well together indeed, and it would be surprising if at this point we found grounds for thinking them “disharmonious”.

We agree. But the more general motivations for the proposed constraints need to be borne in mind before one becomes overly sanguine about the parity of Hume’s principle and the Dedekind-Peano-axioms in the relevant respect. If consistency and conservativeness have to do with the acceptability of the vehicle of an implicit definition as true, generality and harmony have rather to do with its effectiveness in conferring a sufficiently comprehensive and coherent linguistic practice. One may have no doubt that Peano arithmetic represents a sufficiently comprehensive and coherent linguistic practice to count as fully meaningful without thereby granting that the meanings involved could be fully communicated by a would-be implicit definitional stipulation of its second-order axioms.²⁵

Let us return to the issues about arrogance and conditionality. We contend that MacFarlane is gravely mistaken to discount so quickly the idea that there is an important additional constraint in this vicinity. It is true that example J above flouts Conservativeness, but it would be an error—which we do not attribute to MacFarlane—to suppose that Conservativeness will mop up just as well as anti-arrogance in general. To see this, it suffices to adjust the example. Suppose we have it in mind to introduce “Goldie” as a nickname for the smallest counterexample to Goldbach’s conjecture and consider the corresponding pair of stipulations:

G Goldie is the smallest even number which is not the sum of two primes, and
CG If any number is the smallest even number which is not the sum of two primes,
it is Goldie

G is an arrogant stipulation. But it is moot whether it is non-conservative—in order to be so, we will need to suppose it added to a theory which includes the natural

²³ It ceases to be a triviality, of course, when *other* forms of consequence of the introduced form of statement—in this case, a statement of numerical identity configuring canonical numerical terms—besides those directly assigned to it by the elimination rule are taken into account and the requirement is that they too be justifiable on the basis of the premises for the introduction rule. Then, for example, issues have to be confronted like whether $(\exists x)x = Nx:Fx$ is justified on the basis of F’s one–one correspondence to itself, and the issue of harmony becomes inseparable for the question of the acceptability of the abstraction, rather than providing a control upon answers to it.

²⁴ The quotation is from Sect. 2 of MacFarlane’s paper in this volume.

²⁵ For example, one might, quite plausibly, take our grasp of the Dedekind-Peano primitives to be acquired informally through practice with simple arithmetic and counting, rather than conferred via the Dedekind-Peano axioms as implicitly definitional.

numbers in its ontology and which does not entail that there is a counterexample to Goldbach. But if there is such a counterexample, it will be necessary that there is and hence, for a wide class of conceptions of logical consequence, there will be no theory of which this is not a consequence. In that—epistemically possible—case, G will be both conservative and arrogant. In general, conservativeness is a logical property, arrogance an epistemological one. An abstraction may be conservative and yet its stipulation still be arrogant precisely when it is reasonable to demand an independent assurance that certain of its consequences for, say, the ontology of a prior theory are indeed independently accessible within that theory.²⁶ Although we grant that more should be done to make it clearer, we stick to it that anti-arrogance is indeed a crucial supplementary constraint, whose force is captured by none of the other four, singly or in combination.²⁷

Nor, we have to grant, is compliance with it ensured by insisting that implicit definitions take an appropriately conditional form. The reader might be inclined to think that MacFarlane's example of the Pickwick-conditional stipulation of the conjunction of the Dedekind-Peano axioms already makes the point. But to drive it home, consider

C*J If everything is self-identical, Jack the Ripper is the perpetrator of this series of killings

This is no less conditional than CJ, but no less arrogant than J. Mere conditionality of form is thus no assurance of what we want: viz. that in laying down the stipulation in question we merely fix truth- or satisfaction-conditions—merely orchestrate the use of novel vocabulary—making no assumptions about what the world is like in relevant respects.

In essence, a stipulation is arrogant just if there are extant considerations to mandate doubt, or agnosticism, about whether we are *capable* of bringing about truth merely by stipulation in the relevant case. We cannot make it true by stipulation that a single assassin was responsible for the 1890s Whitechapel murders, or that Goldbach has a counterexample. The good implicit definitions are ones where there is no condition to which we commit ourselves in taking the vehicle to be true which we are not justified—either entitled or in possession of sufficient evidence—to take to obtain. Even if there were no doubt about the meaning-conferring credentials of a stipulation of Dedekind-Peano—the matter which will occupy us in the next section—it is the view of the present authors that such a stipulation would be stationed on the wrong side of this line, and that the best abstraction principles, like Hume's Principle, are stationed on the right side of it. More about this in the sequel.

3

The neo Fregean programme for arithmetic aims both to explain the concept of natural number and to provide a means, in the light of that explanation, whereby the fundamental truths of arithmetic can come to be known. The stipulation of Hume

²⁶ This point will be important later—we return to it in Sect. V below.

²⁷ This claim is reinforced by the argument of the Appendix.

is offered as enshrining both the desired explanation of the meaning of the arithmetical primitives and the relevant epistemological (deductive) resources. These two projects—meaning-fixation and knowledge-conferral—are separable.²⁸ An arrogant stipulation, for example, might succeed in the first but fail in the second. More dramatically, it's not implausible to hold that Basic Law V, conceived as a stipulative definition of the course-of-values operator, could succeed, its unsatisfiability notwithstanding, in fixing *some* kind of meaning for that operator—*some* kind of concept of set (course-of-values), which the succeeding generation of workers in foundations at least grasped well enough to try to repair—even though here knowledge is precluded from the start and the traditional connection fails.²⁹ So we do well to take issues about the possible accomplishments of a stipulation of Dedekind-Peano in two stages, enquiring first how well such a stipulation would serve the project of meaning-explanation, and then separately how well it might serve the production of arithmetical knowledge. MacFarlane's suggestion, in effect, is that a stipulation of Dedekind-Peano would appear to do just as well as one of Hume in both respects. Our response here will be concerned merely to highlight respects in which, as it seems to us, the stipulation of Dedekind-Peano does relatively badly—relatively, that is, in comparison with Hume. It is, of course, vital that the considerations to be offered prove appropriately discriminatory if MacFarlane's assessment of the dialectical situation is to be overturned. But whether Hume, more than doing better than Dedekind-Peano, really does do well enough for the neo-Fregean's foundational epistemological purposes is beyond the present discussion.

First, then, on the matter of meaning-determination. How effective would a stipulation of Dedekind-Peano be in determining the meanings of the three arithmetical primitives—"zero", "successor" and "natural number"—which (in standard formulations) they contain? So far we've spoken in a fairly casual way about meaning-fixation, explaining concepts, etc. But now it's important to draw a distinction. The kind of implicit definition in which we're interested in is one whereby, if all goes well, a concept is introduced which is *genuinely novel* in the sense that the language in question otherwise has no means for its expression. It is not, that is, that by laying down the implicit definition, one annexes a new symbol to a concept which one could have expressed independently. In that case, after all, one could better have given an *explicit* definition. Rather, effective implicit definition is to be seen as a means for genuine enlargement of speakers' conceptual repertoire. So our question now is: how well would the stipulation of Dedekind-Peano do in *this* respect?—the respect of explaining the fundamental concepts of arithmetic *ab initio*, as it were?

In order to get the issues into focus, it's useful to compare and contrast a pair of hypothetical stipulations: one of the second order Dedekind-Peano axioms, configuring occurrences of the primitives, "zero", "successor" and "natural number", and one of the so called *Ramsey sentence* of the conjunction of Dedekind-Peano—the sentence arrived at by conjoining the axioms and replacing each of the three primitive arithmetical terms by an appropriate style of variable, and then binding each of these variables

²⁸ This important point is well made by Philip Ebert (2005).

²⁹ The example is Philip Ebert's.

with an appropriate initial existential quantifier. The gist of the Ramsey sentence is thus, roughly, that there is a range of objects which collectively compose an omega-sequence, and each of which occurs at one and only one place in that sequence. Two points are immediately salient about the stipulation of the Ramsey sentence. First, it is not at all—except possibly in the sense we just set aside—a meaning-conferring, or concept-explaining stipulation. For it is expressed entirely in what we are supposing to be *previously understood* vocabulary—the vocabulary of higher-order logic. True, it might be used to explain to someone what an omega-sequence was—to explain the characteristic structure of such a sequence. It would, however, be odd to deploy it for that purpose, which would be more naturally achieved by explaining that an omega-sequence is one of which the Ramsey sentence would be characteristically true, rather than by stipulating that it *is* true.

In short: the fashion in which the stipulation of the Ramsey sentence might communicate a new concept is not that in which MacFarlane and we are interested—the case in which we are interested is a case where a concept is explained for which we previously had no means of expression; and is explained, moreover, by laying down a semantic role for novel vocabulary which does express it. Connectedly, the *stipulation of the truth* of the Ramsey sentence plays no essential role in the explanation of the concept—*omega-sequence*—which one might thereby regard as explained. The explanation would be no less effective if the Ramsey sentence were simply presented as hypothetically characteristic of structures of the relevant kind.

A theorist, then, who proposes that a stipulation of Dedekind-Peano themselves, rather than their conjunctive ‘Ramsification’, can serve as meaning-determining in a way that will allow the proponent of such a stipulation to plug into the epistemology of implicit definition and the traditional connection in the way Neo-Fregean proposes vis-a-vis the stipulation of Hume,—such a theorist has to think that it somehow makes all the difference if the quantifiers and bound variables are removed and replaced by the original three arithmetical primitives. But this claim looks to be extremely tenuous. If it is fair to characterise the stipulation of the Ramsey sentence as, so to say, the issuing of an injunction:

Let there be an omega-sequence!

then it looks as though all that gets added when what is stipulated is not the Ramsification but the second order Dedekind-Peano axioms themselves is the extra content conveyed by the injunction:

Let there be an omega sequence whose first term is *zero*, whose every term has a unique *successor*, and all of whose terms are *natural numbers*!

And the trouble is, evidently, that it is not clear whether there really is any extra content—whether anything genuinely additional is conveyed by the uses within the second injunction of the terms “zero”, “successor” and “natural number”. After all, in grasping the notion of an omega-sequence in the first place, a recipient will have grasped that there will be a unique first member, and a relation of succession. He learns nothing substantial by being told that, in the series whose existence has been stipulated, the first member is called “zero” and the relation of succession is called “successor”—since he does not, to all intents and purposes, know *which* are the objects for whose existence the stipulation is responsible. For the same reason, he learns nothing by

being told that these objects are collectively the “natural numbers”, since he does not know what natural numbers are. Or if he does, it’s no thanks to our stipulation.

Again: if stipulation of the Ramsification of the Dedekind-Peano axioms does not pass muster as the fixing of a new concept in the sense which interests us, then it is not clear how the stipulation of the Dedekind-Peano axiom themselves can do significantly better. What concept can a recipient distinctively and correctly claim to have come to grasp as a result of such a stipulation? Not the concept of an omega sequence, since that’s captured by the Ramsey sentence, which there is in any case no need to stipulate in order to get it across. Not the concept of natural number since one does not, after such a stipulation, know any more about what natural numbers are than before—one merely knows that the natural numbers compose an omega sequence. Not the concept of zero—of course one will know that “zero” denotes the first of the natural numbers when they are arranged in an omega sequence under the “successor” relation; but that isn’t to know much if one doesn’t know what natural numbers are and doesn’t know which of the uncountably many ways of arranging them all into an omega sequence corresponds to the intent of “successor”; and not, finally, what relation is expressed by “successor”, since one doesn’t know which object “zero” denotes. So it is open to question whether the stipulation of Dedekind-Peano would actually be a meaning-determining—concept-constituting—stipulation at all.

The stipulation of Hume fares, we contend, much better. Someone who takes it that Hume is true should take himself to have learned that the referents of the newly introduced terms are invariances under one-one correspondence and hence, whatever else may be true of them, effectively provide a measure of that property of a concept which is fixed by its relationships of one-one correspondence to other concepts—its cardinality. Hume thus contributes a characterisation of the nature of (finite) cardinal number that is unmatched by Dedekind-Peano, which convey no more than the collective structure of the finite cardinals—something which, since it entails those axioms,³⁰ Hume also implicitly conveys. If moreover the stipulation is received as a characterisation of a criterion of identity³¹ for the objects concerned, then the effect (or so we have repeatedly argued, modulo Caesar issues) is to convey a sortal concept of number and thereby to provide the means for basic individuation of particular numbers. By contrast, the stipulation of Dedekind-Peano, even if the vehicle is assumed to be a necessary truth, conveys no conception of the sort of thing that zero and its suite are—they could be anything at all, provided they are countably infinite and (therefore) allow of a serial order.

Here is the score to this point:

The stipulation of Hume serves to communicate a singular-thought-enabling conception of the sort of objects the natural numbers are and explains their essential connection with the measure of cardinality.

³⁰ But see below.

³¹ This involves more, of course, than simply accepting the stipulation as true. For discussion, see [Hale and Wright \(2001a, p. 385\)](#) and following, especially pp. 388–389.

The stipulation of Dedekind-Peano communicates no such conception, and actually adds no real conceptual information to what would be conveyed by a stipulation of their collective Ramsey sentence.

These contentions may provoke the following response.³² Why cannot the argument for the conceptually non-innovative character of the stipulation of Dedekind-Peano be run equally well for Hume's Principle? Take Hume's Principle and Ramsify it. That is, replace every occurrence of the numerical operator with a third-order variable, then existentially quantify. What will the stipulation of Ramsey-Hume communicate? Well, that there is a function from concepts to objects that meets certain conditions, viz. delivers the same values for concepts which are one-one correspondent and different values for those which aren't. Now, the critic may ask, does being told that this function is to be called "the number of" convey any extra content? He may continue, parodying our observations above:

We learn nothing substantial by being told that the function whose existence is here stipulated is called "the number of"—since we do not, to all intents and purposes, know which is the function for whose existence the stipulation is responsible. If stipulation of the Ramsification of Hume's Principle does not pass muster as the fixing of a new concept in the sense which interests us, then it is not clear how the stipulation of Hume's Principle itself can do significantly better. What concept can a recipient distinctively and correctly claim to have come to grasp as a result of such a stipulation? Not the concept of a function that maps equinumerous concepts to the same objects and non-equinumerous concepts to distinct objects, since that's captured by the Ramsey sentence, which there is in any case no need to stipulate in order to get it across. Not the concept of "the number of", since one does not after such a stipulation, know any more about what "the number of" function is than before—one merely knows that it is a function that maps equinumerous concepts to the same objects and non-equinumerous concepts to distinct objects. . . . The stipulation of the Dedekind-Peano axioms fares much better. Someone who takes it that the Dedekind-Peano axioms are true should take himself to have learned that numbers, whatever else may be true of them, form an omega-sequence and hence effectively provide a way of counting arbitrary sequences—a measure of ordinality. The Dedekind-Peano axioms thus contribute a characterisation of the nature of (finite) ordinal number that is unmatched by Finite Hume's Principle, which conveys no more than conditions for identity and non-identity of the finite cardinals—something which, since they entail Finite Hume's Principle,³³ the Dedekind-Peano axioms also implicitly convey.

We are unmoved, however. This is a nice rejoinder provided it is assumed that the stipulation of Hume's Principle can only work by, in effect, stipulating Ramsey-Hume

³² Indeed, have provoked it!—from John MacFarlane in correspondence. The wording following corresponds closely to his communicated formulation.

³³ When a suitable *explicit* definition of "the number of" is provided—see note 44 of MacFarlane's contribution to this volume.

and then annexing “N”, or “#”, or one of the other usual suspects, to the function—indeed, which function?—whose existence is stipulated. But while, in our view, that *is* exactly the way that a stipulation of Dedekind-Peano has to work, it is not at all what is happening with the stipulation of Hume’s Principle. There is no need to (attempt to) stipulate that a suitable function exists. Indeed, the existence of such a function is not a matter of stipulation at all, but a consequence of something known as an effect of the stipulation, viz. Hume’s Principle itself. Our thesis is that the stipulation of Hume’s principle gives sense to the cardinality operator not by annexing it to some unspecified function whose existence we might as well have (attempted to) stipulate directly, but by fixing the truth-conditions of the canonical statements of numerical identity in which it occurs. It is by so bestowing a sense upon the operator that the claim is grounded that the function concerned exists.

In the end, then, the meaning-conferring credentials of Hume’s Principle, and its superiority in this respect to Dedekind-Peano, prove to rest on its knowledge-conferring credentials—on the idea that its stipulation can accomplish what the traditional connection demands, with the vehicle of the implicit definition involved—Hume’s Principle itself—emerging as knowable a priori, in advance of and as a ground for the truth of its Ramsey sentence. There is, in our view, no way of replicating these priorities with a stipulation of Dedekind-Peano: no way whereby we can effectively lay down the axioms first and thereby have meaning so suffuse across the arithmetical primitives that the axioms get to be knowable a priori, and are available to ground their Ramsification. Indeed, so far as we can see, there is no reason to think that a stipulation of Dedekind-Peano axioms achieves an effective implicit definition of any concept at all.

4

The larger issue, then, concerns the capacity of either stipulation somehow to found *knowledge* of the principles—the vehicles of the stipulations—concerned. Since it is, of course, a necessary condition for a stipulation’s producing knowledge that it produce *truth*, an account is owing of what so much as makes it possible for ordinary thinkers—we human beings—to bring truths of the kind in question into being by mere stipulation.

If, as suggested above, it is true that the stipulation of Dedekind-Peano axioms would not, in any substantial sense, confer meaning upon the three arithmetical primitives involved—if the effective content of such a stipulation would be merely that the Ramsification of the conjunction of the axioms of second order arithmetic is to hold—then in this case any knowledge-by-stipulation proposal looks to be far-fetched at best. Before there is any question of anybody’s knowing the vehicle to be true, the stipulation has first to make it true. Regrettably, we human beings are actually pretty limited in this department—in what we can make true simply by saying, and meaning: let it be so! No one can effectively make it true, just by stipulation, that there are exactly 200,473 zebras on the African continent. How is it easier to make it true, just by stipulation, that there is an ω -sequence of (abstract) objects of some so far otherwise unexplained kind? And even if we do somehow have such singular

creationist powers, does anyone have even the slightest evidence for supposing that we do?

It is not that human mental acts have *no* creationist powers. If one takes the view that fiction is created by an act, or series of acts of stipulation, and that the right way of thinking about the characters of fiction is to take them to be a kind of real (albeit fictional) object, then fictional objects will be one kind of creature of stipulation. But the aim here is not a fictionalist account of the foundations of mathematics. Neo-Fregeanism is meant to be at least consistent with the *mind-independence* of mathematical objects.

What, we take it, it is uncontroversial that we can do is to stipulate *meanings*—senses—and *thereby* to stipulate such truths as are involved in doing that. We can stipulate, and in the general run of cases thereby make it true, that a certain predicate is to have a certain satisfaction condition, or a certain sentence a certain truth condition. The vehicle of such a stipulation can properly be a quantified biconditional connecting, for example, an object's satisfaction of the new predicate on the one side with its satisfaction of the cited condition on the other. And even if such a biconditional is not the actual vehicle of the stipulation—even if, for example, we merely propose:

Brother =_{df} male sibling,

—the truth of a corresponding biconditional will be, in the best case, a consequence of the stipulation. Thus, in the best kind of case, we *can* bring it about, just by stipulation, that a biconditional sentence is both understood and true. Once it has been stipulated that “brother” is satisfied by an object just in case it is a male sibling, we have ensured both that “For all x : x is a brother if and only if x is a male sibling” expresses a truth and that it is intelligible. But the meaning-fixation could as well proceed by the stipulation of that very sentence.³⁴

That's the basic template to which the Neo-Fregean proposes to fit Hume and other good abstraction principles. There are, to be sure, many differences between these cases and the kind of explicit definition by citation of a synonym which we've just reviewed. But the neo-Fregean position is that these differences do not disqualify the extension of the basic idea to abstraction principles, at least in the best cases. Abstraction principles, in the best case, are tantamount to legitimate schematic stipulations of truth conditions. Whereas—to repeat a point we have made many times but seemingly must affirm again—to lay down Dedekind-Peano as true is to stipulate, not truth-conditions, but *truth* itself.

All that notwithstanding, MacFarlane may still seem to have a very pointed question: How *can* the direct stipulation of Dedekind-Peano possibly be in a different case? The consideration that makes it seem as though the two stipulations *cannot* be relevantly different is of course the equivalence of Frege arithmetic and Peano arithmetic.

³⁴ Just briefly to revisit the connection between legitimate implicit definition and conditionality of form. We wish to claim no more than this: that as far as what may feasibly be stipulated is concerned, a safe assumption is that we can, in the best case, succeed in stipulating meanings, by way of laying down truth- and satisfaction-conditions; and that the most natural, perhaps inevitable way of doing this is to stipulate (bi)conditionals affirming under what circumstances the conditions in question are met. We do not affirm that conditionality of form provides any assurance that merely this, legitimate and feasible enterprise, is what is going on—indeed the example of stipulations like C*J considered above shows that conditionality as such provides no such assurance.

If the two systems are equivalent, then surely the two stipulations are equivalent? And if the two stipulations are equivalent, how can differential judgements be sustained about their epistemological legitimacy and interest?

The beginning of an answer is that the rhetorical presupposition of the first question—that the equivalence of Frege arithmetic with Peano arithmetic enforces the equivalence of the two stipulations—is a *non sequitur*. Obviously the proof-theoretic output of any axioms is hostage to the inferential machinery in which they are deployed. Consider therefore the respective effects on Frege arithmetic and Peano arithmetic if the regular second-order logic which both utilise is weakened so as to conform with the Aristotelian presupposition that only *instantiated* concepts will be taken to lie within the range of the higher order variables. Thus, in particular, there is no longer any empty concept—every concept has at least one instance. The immediate effect is that Frege arithmetic is crippled: in particular, the proof of the infinity of the number series is undercut, for since there is no empty concept with respect to which to define the number zero, *identical to zero* will no longer be a guaranteed singleton concept, and so on. In an Aristotelian setting, Hume’s Principle will furnish no more numbers than there are individuals in the domain. By contrast, the proof of the infinity of the number series furnished by the Dedekind-Peano axioms is unimpaired by their location within an Aristotelian second-order logic. From a model-theoretic perspective, Hume set in a pure Aristotelian logic has models of every finite cardinality; but the Peano axioms in that setting still demand countably infinite models.³⁵ The least one has to conclude from this disanalogy is that, as a stipulation, Hume is considerably more modest than Dedekind-Peano: the attempted stipulation of the truth of Dedekind-Peano is effectively a stipulation of countable infinity; whereas whether or not Hume carries that consequence is a function of the character of the logic in which it is embedded—and more specifically, a function of aspects of the logic which, one might suppose, are not themselves a matter of stipulation at all but depend on the correct metaphysics of properties and concepts. In precisely this sense, notwithstanding the equivalence—modulo a second-order logic with full impredicative comprehension—of Peano Arithmetic and Frege Arithmetic, the two stipulations do *not* carry the same content.³⁶

This point, however, only partially addresses the question. After all, any higher order logic is going to admit *some* concepts as in good standing, and any such concept, *F*, is presumably, as a matter of logical necessity, going to be self-bijective. An immediate consequence of Hume’s Principle, for each admissible such concept *F*, will be the instance:

(HP^{inst}) $Nx:Fx = Nx:Fx \leftrightarrow F$ one-one corresponds to *F*

³⁵ And indeed remain categorical, since the normal categoricity proofs require no recourse to empty concepts (Thanks here to Øystein Linnebo).

³⁶ The same point could be made by reference to the way that Hume’s Principle and the Dedekind-Peano axioms respectively perform in the setting of a *predicative* second-order logic. Linnebo (2004) shows that the derivation from Hume’s principle of the axiom asserting the existence of a successor for every number cannot succeed without recourse to impredicative instances of Comprehension. This consideration complements that based on Aristotelian logic since both turn on very traditional concerns about the metaphysics of concepts.

So any such concept is going to be credited with a number by the stipulation of Hume's Principle. True, one does not exactly stipulate the instance by stipulating Hume's Principle—it takes a further inference to get to it, and it goes with the intended epistemology of implicit definition that the consequences of stipulations may not be themselves stipulative in character but mandated items of possible knowledge. Still that seems a better consideration when the consequence in question is at least somewhat remote! And we can anyway finesse the issue by considering the stipulation of a schematic version of Hume's Principle in which the stipulated content is exactly that each instance of it is to be true. In that case, we do, it seems, directly stipulate that

(HP^{inst}) $Nx:Fx = Nx:Fx \leftrightarrow F$ one-one corresponds to F

holds. Maybe the left hand side of this is more modest than MacFarlane's mooted stipulation of

(DP^{bicon}) Conjunction {Dedekind-Peano axioms} $\leftrightarrow (\forall x)(x = x)$

But both left-hand sides are substantial and both right-hand-sides are logical truths. If the correct thing to say about the latter is that, its biconditional form notwithstanding, it is tantamount to a stipulation of existence, why isn't that the right thing to say about the former? And if it is, are we not in effect taken back to the dialectical situation as MacFarlane conceived it—with no essential epistemological difference in kind between the stipulations of Hume and of Dedekind-Peano, but only potentially one of degree of strength, depending upon the character of the underlying logic?

This is the heart of the matter. (DP^{bicon}) is, in our view, an arrogant stipulation which there is absolutely no reason to think we are capable of carrying off. (HP^{inst}), by contrast, even though its existential implications are no less immediate or foreseeable than those of (DP^{bicon}), is to be viewed as part of a package of stipulations whose role is to fix the truth-conditions of statements of numerical identity and which are as feasible as any other purely meaning-conferring stipulations. It is because they are this, first and foremost, that there is no need—indeed no room—for any associated stipulation of numerical existence.

The crucial point is one we have elaborated on other recent work, so we offer just an outline of it here. What is going on has a near-enough exact parallel with two ways of thinking about the principles of Comprehension standardly cited in the presentation of systems of higher-order logic. A standard formulation of classical Comprehension provides that, for each formula, F , of the language in n argument places, $n \geq 1$, there is a relation in n places, X , such that for any n -tuple, $\langle x_1, \dots, x_n \rangle$, X holds of $\langle x_1, \dots, x_n \rangle$ just in case " $F(x_1, \dots, x_n)$ " is true. These axioms, which may involve a variety of restrictions on admissible substituends for F , are typically viewed as determining which predicates—open sentences—are associated with an entity belonging to the higher-order 'domain' and are thus safe for generalisation and instantiation; in effect, they are to be compared with postulates in a free first-order logic telling us which of the terms in a language in question have reference to an object. Controversy about classical Comprehension arises because, or so some have felt, there is reason to doubt whether every open sentence in an arbitrary higher-order language is in fact associated with a property (or whatever one takes to constitute the domain of reference for predicates and relational expressions). Aristotelianism, and predicativist views, are

example of this outlook, as well as the more general austere metaphysical tendency which views the ontology of real properties—those that ‘carve Nature at its joints’, in Lewis’s now rather overworked metaphor³⁷—as much *sparser* than the range of significant open sentences in any moderately expressive natural language.

Suppose one inclines to one or another such limitative view, and confronts a proposed satisfaction-conditional definition of the form

x is a—if and only if . . . (x)

where the place marked by the dash is filled by a lexically novel predicate and that marked by the dots is taken by an open sentence in one argument, F, of some complexity, which is associated with well understood satisfaction conditions. From one perspective—when it is viewed simply as the introduction of a piece of shorthand—such a proposed definition is completely uncontroversial and anodyne. But it may seem to become controversial when one learns that the intention is to allow quantification into the place marked by the dash, so that an object’s satisfaction of F is going to count, going right-to-left across the biconditional, and then existentially generalising, as sufficient for the existence of a property. In that case, the intended purely linguistic stipulation embodied in the proposed definition suddenly seems to open itself to a charge of ontological smuggling. If only predicative open sentences denote properties, or if the range of open sentences, even predicative ones, that express properties is sparse, the proposed definition incorporates an existential assumption which is itself no matter for stipulation and about which independent assurance is demanded. In short, it is arrogant.

The situation should seem pretty familiar! But now reflect that there is a kind of metaphysics of properties—often called “abundant”—for which there is absolutely no problem about the proposed definition. On the abundant conception, it suffices for a predicate, or more generally an open sentence, to express a property that it have well-understood satisfaction conditions. That is enough for there to be, in a very intuitive sense, a way things would have to be in order for them to satisfy the expression in question; and on one conception of property, a property is just such a way things would have to be in order to meet a well-determined condition, and is possessed just in case something is that way.³⁸

We just called the abundant view a ‘metaphysics’ of properties. But the label is apt to mislead. No heavy-duty metaphysical or ontological claim is being advanced. The abundant theorist is not conjecturing that in reality, “out there”, there is indeed—as it happens—a vast plenitude of properties perfectly, or almost perfectly, lining up against the possibilities for significant predication (rather as someone might conjecture that there is, universe-wide, as great a variety of life-forms as human beings are capable of describing). Rather he is scaling the notion of a property to match the expressive resources of language—more specifically, the possibilities of meaningful predication. The conception of property being proposed is that of a way things can intelligibly

³⁷ Actually, the metaphor is probably Plato’s—see Franklin (2001).

³⁸ The terminology of abundant and sparse properties originates in Lewis (1986). The general distinction is in Armstrong (1979). See also Bealer (1982) and Swoyer (1996). For a useful overview, see Mellor and Oliver (1997).

be said to be. There really are such ways. Nor are they ‘mere’ creatures of language, since its powers of expression *in principle* are not a creature of language either but determined by whatever the factors are that determine the answer to the question that so fascinated Wittgenstein in the *Tractatus*—the question of what can be said. The existence of an abundant property is something objective; but it is also something for which a complete assurance is provided—without any element of metaphysical risk, or a gap we might try to close by stipulation—by the possibility of an expression with an appropriate satisfaction condition.

If we amalgamate the abundant view as just characterised with Aristotelianism—the view that only instantiated properties are real—we get a view that provides, in relevant respects, a perfect analogy with the neo-Fregean way of thinking about (HP^{inst}) and its ilk. Just as the truth of the right hand side of an instance of Hume’s Principle is required to ensure the existence of the number denoted by its left hand side terms, so now, the truth of the right-hand side of an instance of the comprehension axiom is required before the existence of the property concerned is assured. Neo-Fregean ‘minimalism’ is, in effect, this Aristotelian form of abundance carried over to singular reference and objects. It is the view that good abstractions provide vehicles for legitimate meaning-conferring stipulation, without arrogance or metaphysical hostages, because—like Comprehension axioms as viewed by abundance theory—they involve no ontological adventure; the notion of object, and of the existence of objects, is likewise scaled to match the expressive resources of language. The concept of object being deployed is that of something that may be spoken of by the use of an intelligible singular term, where it suffices for such a term to be intelligible that it feature, in the syntactic role of a singular term, in sentences whose truth-conditions are well explained, and where it suffices for uses of that term to succeed in speaking of something—this is the point of analogy with Aristotelianism about predicates—that the truth-conditions, so assigned, of appropriate (atomic) such sentences are satisfied. In laying down an abstraction principle (of the best kind), there is no more risk, adventure or arrogant presumption in taking it that one succeeds in referring when instances of the right-hand side are true than, from the abundant Aristotelian standpoint, there is in stipulating satisfaction conditions of a well-understood kind for a novel predicate and, when they are satisfied by something, thereby taking it that one has succeeded in defining a property.

It is the underpinning of the procedure of abstraction by this conception of objects and what suffices for reference to them that fundamentally underwrites the disanalogy we claim between the stipulation of (good) abstraction principles and the stipulation of axioms in general. No doubt it merits further scrutiny and explanation. Here, though we must be content to leave the neo-Fregean orientation on the issues raised by MacFarlane in what we offer as a tolerably sharp focus.

Appendix: On an argument of Ebert and Shapiro—the problem of Easy Mathematical Knowledge

In their co-authored contribution to this volume, Philip Ebert and Stewart Shapiro raise what may initially impress as a devastating objection to the whole idea of a

priori knowledge by implicit definition and the ‘traditional connection’ on which, in our version of the view, the epistemology of neo-Fregeanism rests. We think theirs is indeed a very powerful objection—but not to the whole idea. Rather, it bears on the kind of indiscriminate extension of this epistemology implicated in MacFarlane’s suggestion that one might as well stipulate Dedekind-Peano as Hume. The real thrust of the objection, in other words, is to further support the crucial significance of the distinction between the stipulation of (good) abstraction principles and any attempt, in a neo-Hilbertian spirit, to view the status of axioms in general as that of meaning-determining stipulative truths.

Here in outline is the problem. Suppose—for *reductio*—that the Dedekind-Peano axioms could be known, a priori, as a result of their constituting a successful stipulative implicit definition of the arithmetical primitives they contain. And consider instead their stipulation in conjunction with what is in fact a relatively remote arithmetical deductive consequence of them—that the primes are infinite, for example, or any relatively unobvious theorem of arithmetic. Presumably the very same meanings are determined by this augmented stipulation (certainly, the very same class of models is thereby determined). And presumably any relevant virtue possessed by Dedekind-Peano—conservativeness, for example—is closed under deductive consequence. So—whatever those requisite virtues may be—if the one stipulation produces both knowledge and understanding, why doesn’t the other?

This might be called the problem of “easy mathematical knowledge”. While it may need refinement in order to engage certain specific possible versions of the general implicit-definitional proposal, it certainly seems to collide head-on with all broadly *externalist*-flavoured, implicit definition accounts of basic mathematical/logical knowledge: accounts, that is, according to which all that is necessary for knowledge to eventuate from implicit definition is the mere primitive acceptance of its vehicle against a background of the satisfaction of the relevant additional conditions, so that no additional work by the thinker is required—the mere comprehending acceptance of the vehicle is knowledge-conferring just in case the relevant (perhaps quite elaborate) conditions on success/effectiveness are, as it were, ‘externally’ met, and there is no requirement to show that they are met, nor to do any collateral epistemic work at all. Suppose Dedekind-Peano is such a successful, effective implicit definition, conferring certain meanings on its primitives. Presumably the stipulation of $\{PA + Q\}$, where Q is any interestingly unobvious, purely syntactically derivable arithmetical theorem, (and therefore expressible, of course, in terms of the same primitives) confers the same meanings, and is successful/effective if Dedekind-Peano is. So the interesting theorem Q , which one would normally suppose could be known only by an appropriate specific deduction, can be known *by stipulation*, the same way the Dedekind-Peano axioms (allegedly) are. This, which will go for any syntactic consequence of Dedekind-Peano expressible in purely arithmetical terms, is liable to present as a collapse of the whole idea.

One natural response for the neo-Hilbertian to try is to bite the bullet: to concede that, say, Euclid’s Theorem can indeed be known in this way, and then to attempt to restore an essential role for proof by assigning it the task of delivering not knowledge simpliciter but *knowledge that we know*—of putting us in position to claim knowledge. But this is no progress. Euclid’s deduction puts us in position to claim knowledge of

the theorem only if we are in position to claim knowledge of Dedekind-Peano in the first place. If we are, did the implicit definition itself succeed in putting us in this position? If so, we can re-run the argument and proof *still* wants for any essential epistemic role. If not, what is the neo-Hilbertian story about what does?

As stated, we regard the Ebert-Shapiro point as a further powerful argument to discriminate the stipulation of Dedekind-Peano from that of Hume, to the disadvantage of the former—so a further point to make in answer to MacFarlane. Ebert and Shapiro themselves, however, doubt this, because they think a similar problem afflicts the stipulation of Hume or any good abstraction. They have to do a little work to make out this claim. Here is the gist of it.

Let Q be, as before, any interesting arithmetical theorem and consider the purported abstraction:

$$(HP+Q): (\forall F)(\forall G)(N(F) = N(G) \leftrightarrow [F1 \text{---} 1G \ \& \ (\neg Q \rightarrow \forall x(Fx \leftrightarrow Gx))])$$

If $\neg Q$ holds, then the right-hand side collapses into $(\forall x)(Fx \leftrightarrow Gx)$ (since co-extensiveness implies one-one correspondence), and so the whole is the equivalent of Basic law V. So $(HP+Q)$ entails Q . Since $\vartheta(F, G) \ \& \ Q$ is an equivalence relation on F and G whenever $\vartheta(F, G)$ is, $(HP+Q)$ is of the appropriate form for an abstraction. So if its stipulation opens it to knowledge, we once again get a kind of ‘easy knowledge’—admittedly involving a simple deductive routine this time, but the same deduction in all cases—of any arithmetical theorem, Q .

There is an obvious counter: since Q is arithmetical, its embedding into the right-hand side of the abstraction will frustrate the requirement that, at least in basic instances, the concepts deployed on the right-hand side of a good abstraction must be explanatorily prior to that of the abstracts to which it introduces reference. But Ebert and Shapiro anticipate this. In place of Q , introduce instead its Ramsification, Q^* , wherein all distinctively arithmetical vocabulary is replaced by variables and (appropriate levels of) quantification. The resulting principle, $(HP+Q^*)$, will then be a purely logical abstraction—and if it can be known in the manner in which HP allegedly can, would thus be at the service of knowledge of Q^* in accordance with a pattern of deduction which in principle, would extend to knowledge of the Ramsification of any theorem of arithmetic.

So, Ebert and Shapiro contend, the neo-Fregean too has a problem in this neighbourhood: if a good abstraction can be known to be true on the basis of stipulation, so can the Ramsification of any of its deductive consequences, however interesting or remote—not indeed just on the basis of the original stipulation, as with $\{\text{Dedekind-Peano}+Q\}$ but by means of a modified abstraction and a uniform, wholly unpersuasive deductive routine. That seems just as bad a predicament as before.

There is, however, an obviously questionable presupposition in this. It seemed sure, at least from the broadly ‘externalist’ standpoint, that whatever merit as an implicit definition is possessed by Dedekind-Peano is matched by $\{\text{Dedekind-Peano}+Q\}$. That is the point that sticks the neo-Hilbertian with the easy knowability of Q . But the parallel objection runs against the neo-Fregean only if he has no resources to resist the suggestion that $(HP+Q^*)$ is a good abstraction if HP is. And why should anyone think that? More specifically, why should anyone think that who agrees that, as we have

consistently urged, the *avoidance of arrogance* is a crucial constraint on good abstractions and good implicit definitions generally?

To say this is not to say the things that need to be said in order to provide a properly worked through anti-arrogance constraint to factor into a finished account of which the good abstractions are—though we hope that earlier remarks in this discussion contribute to that. But whatever the detail of that account, it will class the Ebert-Shapiro style of rogue abstraction as arrogant and shut it out with the rest of the Bad Company.

References

- Armstrong, D. (1979). *A theory of universals*. Cambridge: Cambridge University Press.
- Bealer, G. (1982). *Quality and concept*. Oxford: Oxford University Press.
- Boolos, G. (1990). The standard of equality of numbers. In G. Boolos (Ed.), *Meaning and method: Essays in honor of Hilary Putnam*. Cambridge: Cambridge University Press. Pages references to the reprint in Boolos (1998).
- Boolos, G. (1998). *Logic, logic and logic*. Cambridge, MA: Harvard University Press.
- Bostock, D. (1974). *Logic and arithmetic: Natural numbers*. Oxford: Clarendon Press.
- Bostock, D. (1979). *Logic and arithmetic: Rational and irrational numbers*. Oxford: Clarendon Press.
- Cook, R. (2001). The state of the economy: Neo-logicism and inflation. *Philosophia Mathematica*, 10(3), 43–66.
- Dummett, M. (1967). Frege, Gottlob. In P. Edwards (Ed.), *The encyclopaedia of philosophy*. London: Macmillan 1967. Reprinted as ‘Frege’s Philosophy’ in Dummett’s *Truth and other enigmas*. London: Duckworth, 1978.
- Dummett, M. (1991). *Frege philosophy of mathematics*. London: Duckworth.
- Dummett, M. (1998). Neo-Fregeans in Bad Company. In M. Schirn (Ed.), *Philosophy of mathematics today*. Oxford: Clarendon Press.
- Ebert, P. (2005). *The context principle and implicit definitions*. University of St. Andrews, Doctoral Dissertation.
- Evans, G. (1982). *The varieties of reference*. Oxford: Clarendon Press.
- Field, H. (1980). *Science without numbers*. Oxford: Blackwell.
- Franklin, L. R. (2001). ‘Plato’s Joints’, available online at <http://www.columbia.edu/~lrf2001/PlatosJoints.doc>
- Frege, G. (1879). *Begriffsschrift*. Halle: L. Nebert; part translated in P. Geach & M. Black, 1952. *Translations from the philosophical writings of Gottlob Frege*. Oxford: Blackwell.
- Frege, G. (1884). *Die Grundlagen der Arithmetik*. Breslau: W. Koebner. Reprinted with English translation by J. L. Austin as *The foundations of arithmetic*. Oxford: Blackwell, 1950.
- Hale, B. (1994). Dummett’s critique of Wright’s attempt to resuscitate Frege. *Philosophia Mathematica*, 2(3), 122–147. Reprinted in Hale and Wright 2001.
- Hale, B. (2000a). Reals by abstraction. *Philosophia Mathematica*, 8(3), 100–123.
- Hale, B. (2000b). Abstraction and set theory. *Notre Dame Journal of Formal Logic*, 41(4), 379–398.
- Hale, B., & Wright, C. (2000). Implicit definition and the a priori. In B. Paul, & P. Christopher (Eds.), *New essays on the a priori*. Oxford: Clarendon Press. Page references are to the reprint in Hale & Wright (2001).
- Hale, B., & Wright, C. (2001a). *The reason’s proper study: Essays towards a neo-Fregean philosophy of mathematics*. Oxford: Clarendon Press.
- Hale, B., & Wright, C. (2001b). ‘To bury Caesar . . .’. In Hale & Wright (2001a), pp. 335–396.
- Hale, B., & Wright, C. (2003). Responses to commentators. *Philosophical Books*, 44(3), 245–263.
- Hale, B., & Wright, C. (forthcoming a). ‘Abstraction and Additional Nature’ forthcoming in *Philosophia Mathematica*.
- Hale, B., & Wright, C. (forthcoming b). ‘The Metaontology of Abstraction’ forthcoming in David Chalmers et al. (Eds.), *Metametaphysics*. Oxford University Press.
- Heck, R. (1997). Finitude and Hume’s principle. *Journal of Philosophical Logic*, 26, 589–561.
- Horwich, P. (1997). Implicit definition, analytic truth and apriori knowledge. *Noûs*, 31, 423–440.

- Horwich, P. (1998). *Meaning*. Oxford: Clarendon Press.
- King, J. C. (1999). Are complex 'That' phrases devices of direct reference? *Noûs*, 33(2), 155–182.
- King, J. C. (2001). *Complex demonstratives: A quantificational account*. MIT Press.
- Lewis, D. (1986). *On the plurality of worlds*. Oxford: Basil Blackwell.
- Linnebo, Ø. (2004). Predicative fragments of Frege arithmetic. *Bulletin of Symbolic Logic*, 10, 153–174.
- Mellor, D. H., & Oliver, A. (1997). Introduction. In *Properties* (pp. 1–33). Oxford: Oxford University Press.
- Rumfitt, I. (2003). Singular terms and arithmetical logicism. *Philosophical Books*, 44(3), 193–219.
- Russell, B. (1905). On denoting. *Mind*, 14, 479–493. Page references are to the reprint in R. C. Marsh (Ed.), *Logic and Knowledge*. London: George Allen & Unwin, 1956.
- Shapiro, S., & Weir, A. (2000). Neo-Logicist logic is not epistemically innocent. *Philosophia Mathematica* (3), 8(2), 160–189.
- Stirton, W. (2003). Caesar invictus. *Philosophia Mathematica*, 11, 285–304.
- Sullivan, P., & Potter, M. (1997). Hale on Caesar. *Philosophia Mathematica*, 5, 135–152.
- Swoyer, C. (1996). Theories of properties: From plenitude to paucity. *Philosophical Perspectives*, 10, 243–264.
- Weir, A. (2003). Neo-Fregeanism: An embarrassment of riches. *Notre Dame Journal of Formal Logic*, 44, 13–48.
- Williamson, T. (2003). Understanding and inference. *Proceedings of the Aristotelian Society, Supp.*, LXXVII, 249–293.
- Williamson, T. (2006). Conceptual truth. *Proceedings of the Aristotelian Society Supp.*, LXXX, 1–41.
- Wright, C. (1983). *Frege's conception of numbers as objects*. Aberdeen: Aberdeen University Press.
- Wright, C. (1997). On the philosophical significance of Frege's Theorem. In H. Richard, Jr. (Ed.), *Language, thought and logic: Essays in honour of Michael Dummett*. Oxford: Clarendon Press. Page references are to the reprint in Hale & Wright (2001).
- Wright, C. (1998a). On the (Harmless) impredicativity of Hume's Principle. In M. Schirn (Ed.), *Philosophy of mathematics today*. Oxford: Clarendon Press. Reprinted in Hale & Wright (2001).
- Wright, C. (1998b). Response to Dummett. In S. Matthias (Ed.), *Philosophy of mathematics today*. Oxford: Clarendon Press. Reprinted in Hale & Wright (2001).