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Why were you initially drawn to the foundations of mathematics and/or the philosophy of mathematics?

My undergraduate degree at Cambridge (and especially part IIB of the then-called Moral Sciences tripos) involved a concentration on formal and philosophical logic, and the philosophies of science and mathematics. It is unsurprising that such a curriculum, taught in a framework that included the inspiring supervision of Casimir Lewy has conditioned my philosophical interests ever since. But there are a number of other factors that have contributed to sustain this focus. It is no accident that very many of the fathers of contemporary analytical philosophy – Descartes, Leibniz, Kant, Berkeley, Frege, Russell and Wittgenstein – as well as great contemporary figures like Quine, Putnam and Dummett – have been intensely interested in the philosophy of mathematics. While the reasons for this are no doubt complex, two considerations seem to me to be of paramount importance.

The first is that, from a philosophical point of view, the very existence of *pure* mathematics, and its immensely fecund, even indispensable role in application to ordinary empirical thought and in science, makes it an especially challenging phenomenon. Mathematics was viewed as the paradigm of knowledge long before mankind had evolved any robust conception of empirical scientific method – to the Greeks indeed, the idea seems to have come quite naturally that the reach of pure mathematical thought might extend to include the most fundamental explanation of the nature of the world. If the modern mind has reserved that project for theoretical physics, it still remains that, according to our ordinary thinking, much can indeed be known about the world by pure mathematical means; indeed, physics itself would hardly be possible without the use of tools developed by the mathematician whose status as instruments of practical discovery is thus implicitly unchallenged. So it is thus a standing challenge to explain how mathematical techniques are at the service of knowledge, both within mathematics and beyond. The mystery deepens when one reflects that mathematical knowledge seems to be knowledge of things that are so non-contingently. And it deepens still further when one factors in its apparent special subject matter – an abstract realm of numbers, sets, points, planes, and functions. How is it possible to know of such things by pure thought? And how can the knowledge thereby gleaned be of relevance to the ordinary physical world?

I think it was the mix of the ready intelligibility of these questions with their difficulty and their evident importance – the sense one has that answers to them will profoundly condition one's conception of much more ramified issues in metaphysics and epistemology – that explains their initial fascination. A crucial additional stimulus in my own case was attending a course of lectures, in my second year as an undergraduate, on Cantorian set theory and experiencing, like so many others, a sense of intellectual awe at the hierarchies of the transfinite that Cantor seemed to have discovered. It was two years later, on reading Wittgenstein's *Remarks on the Foundations of Mathematics* for the first time, that I became aware of both of the possibility of an utterly different take on the matter, and also of how deep reaching are the hostages, in the areas of mind and meaning, of my original awe-struck reactions. It was that realisation, of the philosophical depth of issues in the philosophy of mathematics, which has done most to sustain my interest in the topic.

The second point is connected but consists in a kind of converse. Encountering Michael Dummett's writings towards the end of my time as a PhD student, I became aware of the extent to which fundamental issues and debates in other areas of philosophy often allowed of an especially focused formulation within the philosophy of mathematics, so that it could to some considerable extent be used as a testing bed for general ideas of importance in areas of philosophy quite remote from it. Dummett's own interpretation of the philosophy of the mathematical Intuitionists is an especially forceful example of this, offering as it does one construction of the opposition between broadly realist and anti-realist controversies about all kinds of discourse. A second very important example is provided by the debates about mathematical existence, in which the various protagonisms – platonism, nominalism, reductionism, fictionalism, and so on – have counterparts that engage, once again, in discourse of almost every kind. There is of course no presumption that these various debates permit a uniform resolution, or that the strengths of the various positions in one area of discourse tie in closely with those they have in another. Exploring the differences is part of what makes the philosophy of mathematics such a fruitful starting point.

What examples from your work (or the work of others) illustrate the use of mathematics for philosophy?

Had the question concerned the "use of mathematics *in* philosophy" I would be sceptical that any significant examples could be cited. There are philosophers – recently, and somewhat notoriously, Timothy Williamson, for example[1] – who would welcome an increased use of, for example, model-theoretic techniques in the way that philosophical issues are discussed. But of course the interest of the results yielded by such methods will depend on the extent to which the models explored adequately capture the concepts under philosophical investigation – something whose determination must necessarily fall back on less formal techniques of reflective characterisation. A good example of the point is provided by Church's thesis, that the effectively calculable arithmetical functions are exactly the general recursive ones. General recursiveness is certainly a mathematical notion, and – on the assumption that the thesis is true – we can learn a lot about the nature of effective calculability by mathematical exploration of the notion of general recursiveness. But mathematics cannot teach us that Church's thesis is true. The proposal that it is is an essentially philosophical conjecture, comparing an informal intuitive notion with a technical one, and as such is beyond verification by formal techniques. It is characteristic of philosophical analysis, even when, as in this case, the analysans draws on technical resources, to be informal and conjectural in this way.

When the question concerns the use of mathematics *for* philosophy, a much more expansive assessment is possible. Gödel's incompleteness theorem stimulated a reinvigoration of the debates

[1] See his "Must Do Better" in P. Greenough and M. Lynch, Eds., *Truth and Realism,* Oxford University Press, 2006.

about mechanism in the philosophy of mind. It provided both for sharp formulation of the terms of the debate, and for one famous (widely disbelieved) argument[2] that opponents of mechanism continue to run to this day. John Wiles' resolution, at last, of the status of Fermat's "Last Theorem" is beginning to provoke philosophical discussion of the status of proofs that deploy, seemingly unavoidably, more advanced branches of mathematics in the solution to problems posed by simpler ones. Skolem's theorems put the awe-struck frame of mind, epitomised in the phrase "Cantor's Paradise", under severe philosophical pressure.[3] And, in my own work, the re-discovery of what has come to be known as "Frege's theorem", that the Dedekind-Peano axioms allow of derivation in a system of second order logic with Hume's Principle as sole additional axiom, has enabled us – as might otherwise have been impossible – to focus on at least some of the profound issues in the basic epistemology of logic and elementary arithmetic that would have been very salient much sooner had the contradiction in Frege's system of *Grundgesetze* never been discovered.

Uses of mathematics for regions of philosophy other than the philosophy of mathematics are less salient. But one very nice example is provided by an observation of Stephen Schiffer. James Pryor's influential epistemological 'Dogmatism' offers an account of the confirmation of perceptual judgements by observation of which it is a central feature that, for suitable contents, "P", an experience as of its being the case that P raises the probability of P independently of any presupposition about the subject's collateral information. This proposal promises to sustain a version of G.E. Moore's so called proof of an external world. If my experience as of my having a hand in front of my face unconditionally raises the probability of my actually having a hand, than it does the same for the consequences that there is an external world (since a hand is a material object, existing in space), and that I am not a disembodied brain in a vat. But according to classical probability theory, when two hypotheses, H_1 and H_2, are each consistent with evidence E, then that evidence can raise the probability of one at the expense of the other only in a context in which the respective

[2]Locus classicus: J. R. Lucas, "Mind. Machines and Godel", *Philosophy* 36 (1961) pp. 112-127. Lucas receives impressive support in Roger Penrose's *The Emperor's New Mind*, Oxford University Press 1999.

[3]See the symposium. "Skolem and the Sceptic", by Paul Benacerraf and Crispin Wright. *Proceedings of the Aristotelian Society*. supplementary volume LIX (1985)

prior probabilities, of H_1 on E, and of H_2 on E, reflect the disparity. If the prior probabilities in turn have to be based on evidence, then Priors proposal in epistemology is inconsistent with classical probability theory.[4]

What is the proper role of philosophy of mathematics in relation to logic, foundations of mathematics, the traditional core areas of mathematics, and science?

There is, of course, no reason why the philosophy of mathematics should have a uniform role in relation to these various disciplines and areas of concern. As far as the traditional core areas of mathematics – arithmetic, analysis, set theory and geometry – are concerned, perhaps the simplest way of captioning the principal role of the philosophy of mathematics is to see it as centred on the instance, local to these areas, of what Christopher Peacocke has usefully termed the "integration challenge".[5] The integration challenge raised by any theory, or area of discourse, is to provide an account of the nature of the subject matter concerned which reconciles it with a satisfactory local epistemology – a satisfactory account of the possibilities for knowledge in that area, dovetailing smoothly with what we take to be our actual methods of knowledge acquisition about it.

Integration presents a challenge because, especially in the philosophy of mathematics, the straightforward approach to the ontological issues characteristically gives a rise to epistemological dissatisfaction, and conversely. Paul Benacerraf's epochal "Mathematical Truth" gives vivid expression to this.[6] Platonism about numbers and sets offers an account of the subject matter of mathematics which has attractions when one is preoccupied by its apparently necessary, non-empirical character; but the price one pays, as philosophers from J.S. Mill to Hartry Field have emphasised, is to obscurantise the role of mathematics in scientific theory, and to call in question our competence to know the propositions of pure mathematics that we standardly take ourselves to know. Field's own notorious response[7] is to deny that mathematics *has* a proper

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[4] James Pryor "The Skeptic and the Dogmatist". *Noûs*. Volume 34. Number 4, December 2000, pp. 517-549; Stephen Schiffer, "The Vagaries of Skepticism" *Philosophical Studies* 119 (2004), pp 161–184

[5] Christopher Peacocke, *Being Known*, Oxford, Clarendon Press 1999

[6] *The Journal of Philosophy*. 70, No. 19, pp. 661-679

[7] H. Field, *Science Without Numbers*, Princeton University Press 1980

subject matter, and to recast the issues about its epistemology precisely in the setting of its utility in science. Constructivist, and formalist proposals, though very different in detail, may be viewed as likewise prioritising the demands of a workable epistemology of mathematics. If the subject matter of mathematics could somehow be made out to be the very symbols we manipulate in the course of doing it, or the very proofs we construct, then it might seem as though there should be no problem about explaining how the methods of mathematics engage with its subject matter. The problem is acute. I believe that the most promising extant approach to providing an account of the central theories of classical mathematics which explains the nature of their subject matter, our ability to know the truths of their standard axioms, and how the latter may be a priori yet carry a content fitting them for their standard applications, is the *abstractionist* programme that has been championed by Bob Hale and myself. But I will not attempt to further the defence of that claim here.[8]

In my view, the primary role for the philosophy of mathematics in relation to the foundations of mathematics is highlighted by Wittgenstein's impatient question, "What does mathematics need a foundation *for*?"[9] It was very much in the spirit of the logical atomism of Russell and Wittgenstein to suppose that something deeper had to underlie the, as it were, temporary resting points of classical mathematical theories in their normal axiomatisations. Frege's attempt to uncover the deeper basis having foundered in paradox, it then understandably came to seem that new foundations were wanted urgently – as if the suspension cables of a bridge had been found to be badly corroded, and collapse imminent unless they could be renewed. But we are no longer atomists, and no longer, most of us, subscribe to the idea that mathematical knowledge rests on, or requires, some deep basis which needs to be uncovered. So Wittgenstein's question can seem well taken. A primary concern for philosophy of mathematics should be to determine whether it is.

It will certainly seem so if one subscribes, as many do, to the broadly Quinean picture of the role of mathematics in empirical knowledge supported by writers such as Shapiro.[10] In that case,

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[8] See Bob Hale and Crispin Wright, *The Reason's Proper Study*, Oxford University Press 2001.

[9] *Remarks on the Foundations of Mathematics*, V, 13

[10] Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology*, Oxford University Press, Oxford, 1997

mathematics no more needs a foundation than physics does. It participates, albeit in a specially fundamental role, in the totality of our scientific knowledge, is known in fundamentally the same ways as the propositions of science, and the axioms of its theories stand or fall under empirical pressure in the end, as witness the case of Euclidean geometry. What stands against this, of course, is the traditional picture of mathematics, or at least some mathematics, as both a priori and relatively certain. For those, and there are again many, who incline to this view, the need to provide an account of how mathematics enjoys this status is paramount. Such an account need not necessarily be technical, as it became in the hands of the logicists. But if mathematics is somehow epistemologically exceptional, there is no dodging the philosophical pressure to explain why.

The interplay between debates in the philosophy of mathematics and issues concerning logic has been one of the most interesting features of twentieth century philosophy of mathematics. Frege famously conceived of logic as a codification of the "laws of thought", a body of absolute, normative principles constraining all rational minds. He conceived of this view as standing opposed to psychologism, a loose name for a number of tendencies united in the idea that logic is somehow a merely descriptive science, concerned with the systematisation of human inferential propensities. But the gradual erosion of Frege's perspective through the twentieth century stemmed less from any psychologistic tendency than from a gradually emerging pluralism, a range of standpoints that retain the normativity of logic, for the most part, but have denied its absoluteness. Some of the impetus to this direction has originated, once again, in Quinean views about the alleged revocability of logic under the pressures of empirical theorising. But it is the Dummett-inspired debate between intuitionists and classicists in philosophy of mathematics that provides the most striking example.

For the Dummettian intuitionist, logic is still normative. And there is still such a thing as the *right* logic. But in stark contrast to anything Frege thought, *which* logic is the right one is allowed to vary with the demands of the subject matter to which it is applied. Classical logic is fine for reasoning involving only finite totalities and decidable properties. But revisions are demanded elsewhere. In particular, it is the intuitionist view that the proper philosophical account of the content of statements concerning the infinite demands a non-classical logic in which the laws of double negation elimination and excluded middle fail, and substantial qualifications are demanded to the interdefinability of the quantifiers, and the De Morgan laws. It is an open question to what extent arguments parallel to those of the intuitionists in mathematics may be developed for statements concerning the potentially undecidable future and past, and also for vague discourse. But it was controversy in the philosophy of mathematics that fomented the idea that logic might be revisable in a principled way on grounds other than empirical expediency while still retaining the Fregean conception of logic as essentially normative over correct thought. It is ironic that the classical reaction against this revisionism is largely motivated by the Quinean standpoint, which is quite antithetical to Frege's.

Comment on the interaction between debates in the philosophy of mathematics and issues concerning science, or the philosophy of science, had best be left to those with more than an amateur acquaintance with the latter. But let me close this section by flagging one issue which I would dearly wish to see better researched: the status of the often asserted claim that only classical analysis is fitted to serve the mathematical demands of contemporary physics. For the little that I understand about the issues, no clear reason is apparent to me why this should be so. But it will demand a very able philosopher with a rare grasp both of classical analysis and of various non-standard, including especially intuitionist accounts, together with an extensive grasp of theoretical physics, to set the record straight. Until someone does the orthodoxy that classical mathematics retains essential advantages in point of "simplicity, power, past success and integration with theories in other domains",[11] is likely to remain unchallenged simply because no one knows any better.

[11] These are actually Timothy Williamson's words about classical *logic* (Williamson. Vagueness Routledge 1994. p. 186) but they are perfect for the sentiment about classical mathematics too.

What do you consider the most neglected topics and/or contributions in late 20th century philosophy of mathematics?

The topic perhaps most obviously neglected, since it was so centrally on the agenda during the nineteenth century and before, is continuity. I have in mind specifically the cluster of issues to do with the application of real and functional analysis to the empirical world. The notions of, for example, a continuous change in height, or a continuous rise in temperature, are notions with, it seems, a relatively clear if rough and intuitive empirical content. What is the proper philosophical analysis of this notion, or – should there prove to be a family – notions, and how does it mesh with the various technical notions of continuity developed in classical analysis, non-standard analysis, and intuitionist analysis? Many of the issues here would surely have loomed large in the unpublished – and probably never written – parts of Frege's *Grundgesetze*, informed as it would have been by his constraint that a proper philosophical account of the concepts of a mathematical theory should somehow write in the potential for its canonical applications. Such an account would demand an analysis of the notion of quantity, and an exploration of whether anything could be said about the kinds of variation in magnitude that quantities of different kinds – height, masses, temperatures, directions, and so on – might be capable of displaying independently of the superimposition upon them of a particular mathematical conception. The question highlighted at the end of the preceding section would also belong to this agenda.

It's not completely clear why, this range of issues, crucial to understanding the role of mathematics in empirical science, should have proved so much less interesting to philosophers of mathematics than issues raised by set theory, the limitative theorems of Gödel and Skolem, and broad concerns about mathematical ontology. Frege's failure, before his programme crashed around his ears, to carry it far enough to demand engagement with them may have been a factor; an obsession with the paradoxes, and with the general epistemological and reconstructive issues which they raise, has probably been another.

What are the most important open problems in the philosophy of mathematics and what are the prospects for progress?

I confine attention to those areas of philosophy of mathematics where I have my own greatest investment, and where I think there are realistic chances of progress. First, then, on the philosophy of set theory: we have known since the paradoxes first broke that, if we are to continue to think of sets as in any significant way the objectification of properties, we are going to have to allow that some properties do not determine sets. The insight then required is to determine what it is about those properties that fail to determine sets explains their failure so to do. There are various traditional answers. But the one that comes closest to being intuitively satisfying, to my mind, is Russell's idea that the rogue properties are those which, in the later terminology of Dummett, are *indefinitely extensible*: properties such that purported quantification over all their instances subserves the definition of new instances which, on pain of contradiction, must lie outside the range of the original quantifiers. I believe that we are getting closer to a satisfyingly rigorous characterisation of this notion, and a consequent deepened understanding of why the paradoxes arise.[12]

Finally to the abstractionist programme. For all the inroads made, the relative neglect of two large issues has somewhat compromised the achievements of the programme to date. The first is the challenge of developing an account of the systems of second order logic in which abstractionist theories are characteristically framed to provide a suitable and satisfying alternative to the Quinean idea that they are in effect systems of set theory – a notion lethal to the interest of abstractionist proposals, which if the Quinean view were correct, would collapse into a rather baroque form of set-theoretic foundation. The other challenge is to find some principled distinction between those abstraction principles which, as one assumes is the situation of Hume's principle, are suitable to play a conceptually and epistemologically illuminating role in the foundation of mathematical theories and those – the principles in "bad company" – are not. The latter question has been worked on extensively recently and there are now clear signs

[12]This optimism is based on my experience of co-authoring "All Things Indefinitely Extensible" with Stewart Shapiro, published in Agustín Rayo and Gabriel Uzquiano (eds.). *Absolute Generality*, Oxford University Press, 2006

that, in one of various ways, a stable, positive account may not be far away.[13] Progress on the former will depend on a radical rethink of the nature of quantification and a deeper understanding of the sense in which it is an operation of logic. I believe that ideas tentatively introduced in my own recent work may provide significant steps towards the prerequisite notions.[14]

[13]See Oystein Linnebo, ed., special number of *Synthese* on the Bad Company Problem, forthcoming.

[14]C. Wright, "On Quantifying into Predicate Position: Steps towards a New(tralist Perspective" in Michael Potter, ed., *Mathematical Knowledge*, Oxford University Press 2007