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DISCUSSION

A Reductio Ad Surdum? Field on the Contingency of Mathematical Objects

BOB HALE and CRISPIN WRIGHT

I

Any sceptic about abstract mathematical entities has somehow to come to terms with two facts: that many of the laws of physical science are formulated in ways which involve overt reference to mathematical entities, and that mathematical theory is pervasively applied in the practice of science. Hartry Field's well-known variety of nominalism responds distinctively by finding no fault with the common-sense, platonist *semantics* of mathematical discourse, seeking to show instead that physical theories can be re-formulated without reference to abstract mathematical entities, and that the justifiability of mathematics as a medium of inference within physical theory can be safeguarded without presupposing its truth.

One purely philosophical difficulty for this approach concerns what modal status it should accord to the fundamental nominalist tenet that there are no mathematical entities. The problem is one to which we have, singly and jointly, called attention on several occasions. It was first noted in Hale (1987, Ch. 5) and receives its most detailed and considered formulation to date in our joint "Nominalism and the Contingency of Abstract Objects"(1992).[1] Field (1993, pp. 285-99) has responded in his recent "The Conceptual Contingency of Mathematical Objects", and that response is the topic of this note.

Unfortunately, much of Field's latest discussion is given to sniping at earlier formulations of the difficulty which were explicitly discarded in our (1992), and some confusion may thereby have been caused about the exact focus of the objection we are lodging.[2] While we recognise that clarity will likely be best served not by detailed skirmishing over previous formulations but by concentrating upon what Field has to offer by way of response to our most recent version, it

[1] See also Wright 1988, pp. 425-73, and Hale 1990, pp. 121-44,

[2] In many respects, Field merely repeats—in his own way—what are in essence the same considerations which we ourselves presented as pointing up the need for an improved and more careful statement of the difficulty—though some of our reasons for dissatisfaction with the earlier versions are distinct from the complaints Field makes against them.

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seems desirable to make one further attempt to dispel certain persisting misunderstandings of the objection and to make clear its intended force. So in the next section we provide a brief review of the original objection and of its subsequent refinements, together with the reasons for them, culminating in a fresh statement of the most recent formulation. §§III and IV will then repel two new counter-attacks which Field makes in his latest paper.

II The Contingency Problem: overview

According to Field, mathematics does not need to be true to be good.[3] The property good mathematical theories must have, he argues, is not truth but *conservativeness*—where a mathematical theory *S* is conservative just in case, when combined with any collection *N* of nominalistically formulated statements, *S* yields no nominalistically formulated conclusions which do not follow from *N* by itself. Now, conservativeness in this sense is very close to consistency, in terms of which it can be defined, and which it in any case requires. If asked how the root notion of consistency is to be understood, the good nominalist must, it seems clear, proceed with some caution. It appears that he cannot gloss this notion in either of the standard ways, in model- or proof-theoretic terms, since both involve unwanted talk of sets of one kind or another. Naturally, Field appreciates this, and proposes instead to take a certain notion of logical possibility as *primitive*. The claim that a mathematical theory *S* is consistent is then to be understood as the claim that it is, in this primitive sense, *possible* for its axioms to be collectively true. Since *S*'s axioms will, in any interesting case, entail that there are numbers of some sort, or sets, or the like, this requires that the existence of such things should be at least possible. The upshot is accordingly that Field must regard their existence, or non-existence, as a matter of contingency—there are, in his view, no natural numbers, for example, but there is no purely conceptual obstacle to the existence of such things.[4]

[3] This snappy formulation is Field's own, from p. 58 of Field 1982, pp. 45-69, and reprinted in Field 1989, pp. 53-78; and also Field 1980, p. 5, 7-16.

[4] We here gloss over a skirmish. A proponent of Field's position may object to the conclusion drawn in the text, as Field himself has been wont to do, on the ground that it involves an equivocation: that it confuses the *logical* possibility—in Field's favoured sense of the term—of the supposition that numbers exist with its *conceptual* possibility; where a statement is conceptually possible just in case, roughly, its negation is not analytic in the broadest sense of "analytic"—i.e. not "true just in virtue of its meaning"—and conceptually contingent just in case both it and its negation are conceptually possible. The complaint is that, while Field is committed to the logical possibility and hence—since in his view there are no numbers—logical contingency of the existence of numbers, logical contingency does not entail conceptual contingency; so it is just an error to attempt to saddle him with whatever awkwardness may attend the idea that the existence/non-existence of mathematical objects is a conceptual contingency.

However, since all uninstantiated logical contingencies must presumably be viewed either as conceptually contingent or as conceptually impossible, there is, as we stressed in

That is apt to seem at least bizarre. But work is clearly wanted to isolate precisely what is exceptionable about it. Accordingly, we now chart the essential moves in our successive attempts to precipitate a clear objection.

A natural initial thought is that if the existence, or non-existence, of numbers is to be a contingency—if, in particular, it is to be a contingent fact that there are no numbers—then it ought to be possible to say something about why the contingency is resolved in the direction in which, on Field's view, it is. There should be some story to be told—and told in terms acceptable to the nominalist—about why there are no numbers, and about how the world would have to have been otherwise, were there to be such things. Yet there appears no prospect of sensible answers to these questions. That invites the suspicion that the notion of contingency is here being misapplied.

So formulated[5], the objection is suggestive at best. For it relies upon a claimed connection between contingency and explanation—a connection which might be captured in a principle to which there are clear exceptions:

(a) It is a contingent fact that *p* only if there is an explanation why it is the case that *p*.

Consider, for instance, the holding of any fundamental physical law, or the existence of certain fundamental particles. There is evidently nothing intrinsically amiss in the idea of things existing, or being thus and so, as a matter of *brute* fact, in the sense that there is no explanation to be had for their existence or character. The possibility of such *brute contingencies* entails that there can be no hard and fast requirement that every contingency actually be explicable, so any straightforward inference from absence of explanation to non-contingency is immediately forestalled.

On the other hand, the suggestion that the target case—the existence or otherwise of numbers, or other abstracta—can be a matter of brute contingency in that kind of way seems utterly implausible. To accept that the holding of certain physical laws, or the existence of certain particles, are *fundamental* matters is, in effect, explicitly to foreclose on the possibility of further explanation of them.

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our (1992), little point to this rejoinder unless it is Field's view that the existence of numbers is a conceptual impossibility—or, more radically, that no workable notions of conceptual possibility and impossibility exist. We argued in our (1992) that neither of these responses is sustainable. And indeed Field himself now declines them both, writing at the outset of his most recent discussion that:

> ...on the most obvious construals of "conceptually necessary" (and "conceptually possible" etc.),...the right view...is that it is conceptually contingent whether mathematical entities exist.

Field should thus be regarded as effectively committed to the conceptual contingency of numbers and other abstract mathematical objects. We return to the charge of equivocation in note 8 below.

"Contingent" and its cognates should be taken to denote conceptual contingency throughout what follows unless there is explicit indication to the contrary.

[5] This formulation corresponds pretty closely to those given in our first presentations of the objection, for which see Hale 1987, pp. 109-112 and Wright 1988, pp. 464-5.

But there seems to be no comparably explicit preparation in the intuitive understanding of the concept of number for the idea that instances of it, if any exist, do so as a brute contingency. This encourages the thought that closer scrutiny may disclose a more modest, but still substantial, principle which maintains a more refined tie between responsible application of the notion of contingency and the possibility of explanation, and against which any attempt to regard the existence/non-existence of numbers as a contingent matter will still offend.

The obvious proposal would be that any bona fide contingency requires, not that an explanation why it is resolved as it is should actually be available, but rather that there should at least be an appropriate species—a *category*—of explanation which may go uninstantiated in the particular case but which a satisfactory *explanans* could in principle instantiate. Even fundamental physical laws *might* have allowed of explanation: there is conceptual space for their explanation—space which happens to be empty but which would be filled, for instance, were appropriate but yet more fundamental laws to be in operation. The objection would then be that in the case which concerns us, there is not even such a category: there is simply no conceptual space for an explanation why there are (aren't) abstract mathematical objects. We have no conception of what the existence of such objects could, even in principle, be contingent on.

Again, though, the general principle on which the objection now relies, viz.

(b) It is contingent that *p* only if there *could*[6] be an explanation why *p*

is unacceptable without qualification. The trouble this time is that it simply legislates against the possibility of what may be termed *superbrute contingencies*—contingencies which are not merely contingently inexplicable but of which all possibility of explanation is a priori foreclosed.[7] Plausible examples of super-

[6] That is, there is at least conceptual space for an explanation why *p*, even if there is as it happens no correct explanation to fill it.

[7] Field's own original response (1989, pp. 43-5) to our first formulations of the objection, re-iterated in his (1993) in relation to what is, in effect, the second formulation given here, is quite different. His contention was that the objection fails through an *equivocation* on the notion of contingency. He grants the intelligibility of "richer" notions of contingency for which it is required, if it is to be a contingent truth that *p*, that there be available some explanation why *p* rather than not. But he claims that we illicitly slide to the assumption that he holds it to be contingently true that there are no numbers in some such richer sense from the fact that he does indeed hold this to be contingently so in an austere logical sense—a sense whereby it is logically contingent that *p* just when it is neither first-order logically true that *p* nor first-order logically false that *p*. (Actually, that is a slightly more restrictive notion than the one Field endorses, but the differences do no affect the issue here.) He now invites anyone who doubts that the objection equivocates to consider what he bills as "the analogous argument in theology", which he formulates as follows:

> Surely the existence of God is logically consistent, so if there is no God, it is *contingently* false that there is a God. But the atheist has no prospect of an account of what this alleged contingency is contingent *on*. There is no explanation of why the world contains no God, and if it had contained one, there would have been no explanation of that either. There are no conditions favourable for an emergence of God, and no conditions that prevent His emergence. (1993, p. 291)

If it may be inferred from this that the proposition that God exists cannot be contingently false, then it follows from its consistency that it must be true. (This reasoning anticipates brute contingency are not hard to conjure. One we gave before is the Ageless Conundrum—"Why is there something, rather than nothing?" This is an unanswerable question if physical existence is at issue and a satisfactory explanation why a physical state of affairs obtains has to advert to a causally antecedent situation in which it does not obtain. For a physically empty world would presumably have no causal progeny. Yet the existence of a physical world surely cannot be conceived as other than a contingency. Another example was the Great Contingency—the global state of affairs that verifies all actually true contingent propositions. Since the obtaining of each of its ingredients is a matter of contingency, the exact character of that global state is itself a contingent matter. But there can be no explaining it—at least not if contingencies can be explained only by reference to *further contingencies*. For there are, precisely, no further contingencies to appeal to; all else is a matter of necessity.

Field's later argument about *surdons*—see §IV below.)

Commenting that this is "in effect what is known as Anselm's second ontological argument", Field takes it for granted that we would reject *this* argument, at least, as vitiated by an equivocation on the notion of contingency—a slide from the logical contingency of God's non-existence to some such claim as that it is contingent in the sense that alternative historical developments might have brought Him into existence—and complains that he cannot see how the case of numbers is relevantly different. (loc. cit., pp. 291-2)

This is as good a place as any to correct a number of misunderstandings and apparent misreadings. The first point to observe is that Field has, of course, no good reason to assume that we must either convict this version of the Ontological Argument of equivocation or else accept it. We need do neither, since the considerations about to be displayed in the main text—which, by the way, are explicit in our (1992): see §IV, especially p. 127—show that the principle (b) on which that argument relies is anyway faulty, so that there are ample grounds to reject the argument without reliance upon spurious charges of equivocation.

Secondly, it is exceedingly difficult to understand Field's reiterating the charge of equivocation *after* his admission that the existence of mathematical entities is a conceptually contingent matter. Is it that he thinks that the objection conflates not logical but conceptual contingency with some yet richer notion? We can only guess. But in any case, as stressed at pp. 117-8 of our (1992), the whole idea that something other than Field's austere notion of logical contingency must be in play, if there is to be a prima facie legitimate demand for explanation why there are no numbers, given that that is at most logically contingently so, was quite misbegotten in the first place. On the contrary, the vast majority of logical contingencies—in Field's sense—consist in the obtaining of states of affairs for which we should expect an explanation to be, in principle anyway, available. That is quite enough to get the question "why?" off the ground; if a particular (type of) case is to be excepted, then grounds for excepting it are called for. Field seems repeatedly to have overlooked the difference between claiming that the existence of an explanation is entailed by an assertion of contingency, and claiming that a request for an explanation is presumptively in order (even if defeasible in special cases).

Finally, Field has no business ascribing to us any attempt to prove (in the passage to which he is reacting) the existence of numbers by a variant of Anselm's argument—or, for that matter, in any other way. While we do, of course, hold the existence of numbers to be a matter of conceptual necessity, the argument Field is discussing was not a positive argument for platonism, but a *purely negative* one, directed at his own peculiar brand of nominalism, along with any other view which requires the existence of numbers to be a contingent matter. Endorsement of our argument, as such, involves *no commitment* to platonism—if the argument were good, it would be equally at the service of, for example, any nominalist of a more traditional, reductive stripe. In short, the Contingency Objection is just that—an objection: it is not, and never was intended as, a self-standing argument for platonism—as, indeed, we have previously emphasized (1992, cf. pp. 134-5 and fn. 27).

To one in sympathy with the basic line of objection, however, the failure of principle (b) is merely likely to suggest the direction in which an improved formulation should run. The basic intuitive thought behind the objection is that, for any recognisable notion of contingency, the claim that it is contingently false that *p* generates a legitimate but *defeasible* demand for an explanation why it is not the case that *p*. What the cited counter-examples to (b) illustrate are certain ways in which that demand can be legitimately defeated. They show how, quite consistently with the correctness of a robust intuition of contingency, the nature of their subject-matter and/or of all relevant kinds of explanation may rule out a priori all possibility of explanation of certain contingent truths. That the Ageless Conundrum and the Great Contingency are inexplicable is in each case a direct consequence of the very conception of the putative explanandum, together with an appreciation of what types of explanation, if any, could be appropriate. But the existence of such cases does nothing to deny the *presumptive* connection between contingency and the possibility of explanation, nor to excuse a believer in an apparently inexplicable contingency the work of defeating the presumption. Yet there is in view no plausible ground, emanating from the concepts and subject-matter involved, for thinking that it must *both* be a contingency *and* a constitutionally inexplicable one whether or not there are numbers.

This may suggest that the objection might profitably be developed by relying on something like the following principle:

> (c) It is contingent that *p* only if *either* there could be an explanation why *p* or there is available a priori[8] an explanation why the (putative) fact that *p* must resist explanation[9].

In effect, this rules that no bona fide contingency can be *both superbrute* and *yet inexplicably so*. Obviously any merely brute contingency complies. And if the examples thus far considered are representative, so do superbrute contingencies. On the other hand, it is plausible that the putative contingency that there are no numbers violates it. For we seem essentially to lack both any conception of what might constitute a satisfactory explanation of the putatively contingent fact that there are no numbers and any clear account, comparable to those on offer in the cases of the Conundrum and the Great Contingency, of why no explanation can be forthcoming.

We foresee a doubt about the stability of the last claim. Would not cogent argument for its first component—that no conception exists of what might constitute a satisfactory explanation of the non-existence of numbers—be bound to provide,

[8] Available *a priori* because we are concerned, in the second disjunct, with putative contingencies which are more than merely brute; that is, contingencies in regard to which there is not even conceptual space for an explanation. In such cases, an explanation for the unavailability of an explanation why *p* will, presumably, have to draw upon broadly conceptual considerations—concerning the content of the putative contingency, for example, or the type of explanation, if any, which would be appropriate—and thus be available a priori.

[9] For ease of exposition, we have offered here a somewhat simplified version of the principle discussed in our (1992) at p. 130 ff.

in effect, what the second component claims we lack: an explanation, comparable to that available in the Conundrum and Great Contingency cases, of why there is no such conception? "Why are Hale and Wright so confident that we have no such conception unless *something* in the concept of number can be taken to preclude such an explanation, and hence to explain why no explanation can be given?"

We agree that it has to be possible to say more about the unavailability which we allege of any category of explanation in the target case. However, the more that has to be said will provide no comfort for the nominalist unless, as with the Conundrum and the Great Contingency, it at least squares with—or better, supports—the claim of contingency in the first place. There is no comfort for the nominalist if the best account why the second disjunct of (c) is satisfied proceeds by reference to features—or claimed features—of the concept of number which belong with the opposed platonist conception, according to which the existence of numbers is no contingency but a necessity. That of course is the kind of account which *we* would aim to offer.

Nevertheless, we declined to press the (c)-based objection. Our reluctance to do so had its source in the following train of thought. Let C be the statement of any *merely* brute contingency, and let C^* be the statement that there is no explanation why C. Then C^* is true, and must be at most contingently so (else C would be superbrute). Could it in turn be a merely brute contingency that C^*? Well, for that to be so, there would have to be the possibility, in principle, of an explanation why C^*. But any purely a priori explanation of C^* would, it seems, establish too much—viz. that there *could not* be an explanation why C, contradicting the hypothesis that C's contingency is merely brute. Yet it is, on the other hand, quite unclear how there could be an *empirical* explanation of the (contingent) absence of all empirical explanation why C. These considerations, though evidently not decisive, encourage the thought that C^*'s contingency must be superbrute. But if that is so, then C^* counter-exemplifies (c), unless it can be explained, a priori, why no explanation why C^* is to be had—unless, that is, it can be explained a priori why there is no explanation why there is no explanation why C. And here again, it is apt to seem quite obscure how the required explanation might run. (Cf. Hale and Wright 1992, pp. 131-2.)

This difficulty is evidently less than conclusive; it has not been demonstrated that C^* is a counter-example to (c), and we do not exclude the possibility of a demonstration that it is not: But the problem is that the claim of C^* to contingency seems perfectly secure nonetheless: that C^* *is* clearly contingent even though it is not at this point clear that it complies with (c). So it must be regarded as an open question whether principle (c), or some near relative, can be sustained in full generality. Accordingly, while—to stress—it is still possible that a version of the objection proceeding from (c) might eventually be upheld, it was at this point that the development in our (1992) took a somewhat different tack.

Hitherto, we have focused upon the idea that, typically, genuine contingencies are *contingent on* something, and sought to disclose constraints on exceptions. The vast majority of contingencies, however, not only depend upon something (other contingencies) but have things (further contingencies) *depending upon them*. Let us say that a contingency is *barren* if there are no other contingencies whose obtaining depends on it. Uncontroversial instances of barren contingency are none too easy to come by. However, the occurrence of mental events, as conceived by epiphenomenalism, might be one kind of example (and we are not, in any case, about to rehearse for barrenness analogues of the preceding considerations about bruteness.)[10] The striking point is rather as follows. While we can reasonably readily conceive of a range of brute (and even superbrute) contingencies, and can—though perhaps with rather more difficulty— muster examples of barren ones, there appear to be no independently credible examples of absolutely insular contingency (i.e. of contingencies both brute and barren). Yet the alleged contingent non-existence of numbers would seem to have to be precisely that—*absolutely insular*. Not only is there nothing illuminating to be said—supposing there to be no number 17—about how things would have to have been otherwise in order for there to be such a number, but equally—at least if Field's programme is feasible: if arithmetic is indeed conservative and mention of numbers is everywhere dispensable in the formulation of physical theory—there appear to be no uncontroversial contingencies whose resolution turns, however indirectly, upon whether or not 17 exists.

It may now seem that we are finally poised to argue, via the principle:

(3) There are no absolutely insular conceptual contingencies[11]

to the conclusion that whether or not there are numbers can be no contingent matter. And that, indeed, is exactly how Field interprets us.[12] So it is important to grasp that that is *still* not—or not exactly—the objection. We do think that principle (3), proscribing absolutely insular contingency, has the ring of truth. But we were, and still are, vividly aware that, whilst the denial of the principle unquestionably jars with an exceedingly natural and plausible conception of the realm of contingency as forming a single integrated system, nothing has been put on the table that might be regarded as independent argument for it. Our point, then, is not that Field's nominalism is to be rejected as in conflict with a principle which indisputably governs the notion of contingency; it is rather that, however exactly the question whether there can be absolutely insular contingencies is to be resolved, it is unacceptable that its resolution should be effected, unforeseen and unmotivated, merely by a nominalist philosophy of mathematics. It is not that Field's view is inconsistent with something clearly correct, but rather that it unwarrantedly prejudices the verdict on something metaphysically appealing. Field has all along owed, and after his most recent

[10] We are not, it should perhaps be stressed, ruling out the possibility that further consideration of the notion of barren contingency may disclose constraints upon attribution of *superbarren* contingency which could be seen to be violated in the case that concerns us. But this was not the direction we took in the paper to which Field is responding, so we shall not explore it further here.

[11] We use "(3)" rather than "(d)" to match Field's own nomenclature—see §IV below.

[12] Cf. Field 1993, p. 296.

discussion still owes, an explanation of why it is acceptable for his view to have this consequence—an explanation of why exceptions to (3) are to be expected, and of why mathematical objects in particular may tolerably be taken to be such exceptions.

III A new Ontological Argument?

We turn now to Field's counter-attacks. Field's response to Fregean platonism has been marked by repeated attempts to discredit the Fregean's reasoning to the existence of abstract objects by an alleged association with the Ontological Argument for the existence of God.[13] So it comes as no surprise to find him trying the point again in his latest paper.[14] Rather than make a comparison with any traditional version, however, Field charges this time that Fregean platonism provides the resources for a *new* kind of Ontological Argument—one quite unlike any that occurred to Anselm, and to which the Fregean has no means to object.

The central plank of this argument is the equivalence:

(G): The God that caused x = the God that caused y iff x and y are spatio-temporally related.

Since (G) may be laid down as (partially) explanatory of the concept of God, Field asserts, the Fregean ought to accord it a similar status to that he is willing to grant to the principle $N^=$, that the number of *F*s is identical to the number of *G*s if and only if there are exactly as many *F*s as *G*s, and to the analogous principle for directions, that the direction of line *a* is identical to the direction of line *b* if and only if line *a* is parallel with line *b*—and is therefore in no position to object to the resulting derivation of the existence of the God that caused x for any given spatio-temporal object x.[15] The sane view of the matter, Field contends, is that (G) is a conceptual truth only insofar as God, *if He exists*, must be the final cause of each of any pair of spatio-temporally related objects. But then—in Field's view—we ought correspondingly to claim conceptual truth not for $N^=$, but only for the analogous conditional: *if numbers exist*, then they are identical, or distinct, as $N^=$ dictates.

Say that an equivalence of the appropriate kind—one linking an identity statement concerning items of a certain kind with the holding of an equivalence relation (other than identity) among objects of another (not necessarily distinct) kind—is *non-Fregean* just in case it is a conceptual truth only if understood conditionally, à la Field, and *Fregean* if it is, unconditionally, a conceptual truth. Field of course recognises no such equivalences as Fregean. But it is no part of

[13] See, for example, Field 1989, p. 43 fn. 32.

[14] Field 1993, pp. 286-7.

[15] The derivation proceeds from the relexivity of spatio-temporal relatedness to the claim, for any spatio-temporally situated x, that the God that causes x = the God that caused x, and thence via existential generalisation to the classic conclusion.

our brief now to argue to the contrary. The challenge his example poses is rather that we provide an explanation of how someone *antecedently* disposed to view e.g. $N^=$ and the principle of identity of directions as Fregean, can coherently take a contrary view of (G).

It will help to reflect on a nice example of Michael Potter's:

(MP): *a*'s Member of Parliament is *b*'s Member of Parliament iff *a* and *b* are co-constituents (live in the same parliamentary constituency).[16]

(MP) is a principle about which Field's conditional view seems eminently reasonable. There surely is a conceptual connection between co-constituency and the having of the same member of Parliament. But the co-constituency of *a* and *b* cannot provide a guarantee—let alone a purely conceptual guarantee—that the left-hand-side of (MP) is true (or if it does, then "Members of Parliament" are not human beings).[17] For a parliamentary constituency may be temporarily unrepresented. What is conceptually true is that people who live in the same constituency have the same MP—*if they have any.*

The Fregean will be saddled with the new ontological argument only if he can offer no principled distinction between cases where he proposes to allow the reflexivity of the relation on the right-hand-side to settle the question of the existence of the corresponding left-hand-side objects and cases—like (MP)—where Field's conditional view is surely the correct one. The most salient contrast is that (MP) is woefully inadequate as an explanation of the concept of a member of parliament, since it omits—inter alia—all mention of the consideration that to be a member of parliament is to be a *person* elected to discharge a certain office. The singular terms which feature on the left-hand-side of instances of (MP) are not properly understood unless it is grasped that their reference, if any, is to objects—people—of a kind with which anyone who understands (MP) must already be familiar. No such presupposition attends $N^=$ and the principle of identity of directions. It is, roughly, because the MP-terms have to seek their referents among objects of a kind of which we already have an independent conception that there is space for the possibility of a gap between the holding of an instance of the right-hand-side of (MP) and the existence of the items demanded by the corresponding instance of the left-hand-side.

More generally, consider any equivalence of the appropriate kind:

For all a and b: $S(a) = S(b)$ iff $a \mathrel{R} b$.

The essence of the Fregean view of such a principle is that the truth-conditions of its two sides are held *stipulatively identical*, and that an understanding of the left-hand-side is to be derived under the aegis of that stipulation via a face-value construal of its syntax and vocabulary and a proper understanding of the right-hand-side. So much is precisely what is involved in the idea that the two sides "carve

[16] Potter had his own subversive purposes in mind with this. It would take us too far afield to go into them, and how we would respond to them, here.

[17] As one may independently have begun to suspect!

up" one and the same content (*Inhalt*) in different ways.[18] Accordingly, when a Fregean view is taken of the equivalence, the occurrences of "S(*a*)" and "S(*b*)" have exactly the meaning as is bestowed upon them by their introduction under these constraints—*and no more*. It follows that in any case, like that of (MP), where a proper understanding of the left-hand-side demands a *richer* conception of the meaning of the S-terms than could be gleaned from the stipulation of the equivalence and a face-value construal of the syntax and vocabulary of its left-hand-side, any coincidence in the truth-conditions of the two sides—if indeed their truth-conditions do coincide—cannot be purely stipulative.

The upshot is that a Fregean has indeed no cause to object to the stipulation of (MP) as a Fregean equivalence, from which the existence of referents for the terms on its left-hand-side will follow in any case where the corresponding right-hand-side is true. But such a stipulation will not provide the means for proof, subject to that condition, that Members of Parliament exist. It will not do so because the interpretation of the left-hand-side necessary if its truth is to carry that significance—the interpretation which has it that to be a Member of Parliament is to be a person elected to discharge a certain office—is not an interpretation at which one can arrive by holding the two-sides to be stipulatively equivalent and reading the left-hand-side in the fashion dictated by that stipulation and its surface syntax and vocabulary. The interpretation arrived at in that way does nothing to distinguish MPs from abstracta.

Similar considerations apply to (G). There is nothing in a Fregean reading of (G) to ensure even that the occurrences of "caused" on its left-hand-side are to carry their usual significance. In any case, there is presumably much more to the intended understanding of the singular terms on its left-hand-side—at least if the new ontological argument is to seem disquieting—than is encompassed in the notion: the prime cause of anything spatio-temporal. Moreover the additional features—moral perfection, omniscience, and so on—which the disquieting reading will tacitly include, will number some which might fail to be instantiated by something which was in fact the prime cause of all spatio-temporally related items. In short, (G) can legitimately be treated as Fregean—and the truth-conditions of its two sides held to be stipulatively coincident, just so long as a fully competent understanding of the terms on its left-hand-side—"The God that caused *a*". etc.—can be possessed in innocence of the general notions of causation, of a prime cause of a given range of items, and of God's distinctive perfections. An "ontological argument" whose conclusion carries none of that significance is nothing to get excited about. Conversely, as soon as the left-hand-

[18] Cf. Frege 1884, §64:

> The judgement "line *a* is parallel to line *b*", or, in symbols, *a* ll *b*, can be taken as an identity. If we do this, we obtain the concept of direction, and say: "the direction of line *a* is identical with the direction of line *b*". Thus we replace the symbol ll by the more generic symbol =, through removing what is specific in the content of the former and dividing it between *a* and *b*. We carve up the content in a different way. And this yields us a new concept.

side is interpreted as Field intends, it ceases to be read in a fashion consonant with the Fregean reading, whereby its coincidence in truth-conditions with the right-hand-side can be held to be a matter of stipulation. Rather, a lemma will then be wanted—a link will need to be effected between two contents which are given independently. Field knows no better than the Fregean does how to construct that link.

IV Surdons

Apart from the repeated but misplaced charge that it involves an equivocation on the notion of contingency, Field offers no response to the final version of the contingency objection summarised earlier save in the last few pages of his discussion, in the argument concerning *surdons*. Even here his response rests, initially at least, on a misunderstanding. Recall that a fact is absolutely insular if it is not because something else is so that it is so, and if nothing else is so because it is so. In the final part of his paper, it emerges that Field assumes that the objection we are lodging relies upon the principle that

(3) There are no absolutely insular conceptual contingencies.

He responds that, while he is himself no friend of the absolutely insular, (3) is demonstrably the wrong "anti-insularity" principle to have, since it yields what he regards as absurd consequences, or at least—if someone does not regard the relevant consequences as absurd—yields those consequences in an absurd way.

This, as remarked earlier, involves a misreading. Our objection was not that Field's view of the (putative) fact that there are no numbers infringes an evidently acceptable principle, but that it forces an unmotivated decision against an attractive principle. Nevertheless, the objection wishes, if not to endorse (3), then at least to leave it in play. So if Field can disclose telling considerations against it, that will constitute an effective rejoinder not just to the objection he mistakenly takes us to be making but to the—we hope—rather more judicious objection we actually intend.

What is the alleged problem with (3)? Field writes:

> Call something a surdon iff
>
> (A) its existence and state are in no way dependent on the existence and state of anything else; and
>
> (B) the existence and state of nothing else are in any way dependent on the existence and state of it.
>
> This certainly seems to be a conceptually consistent concept; but (A) and (B) guarantee insularity; so principle (3) immediately guarantees the existence of surdons—indeed, their necessary existence. Of course, Hale and Wright accept this conclusion, since they take numbers to be surdons, but even they should balk at the idea that establishing the existence of mathematical entities is as easy as this. (1993, pp. 296-7)

Field sees this as a *reductio* of (3). We shall argue that that is not the right way of looking at the matter, and that, as a self-standing argument, the line of thought he sketches is indeed suspect for reasons that a Fregean platonist is at no disadvantage to acknowledge.

We take it that the "immediate guarantee" of the (necessary) existence of surdons is issued like this. The definition of "surdon" yields that the existence of surdons, if they exist, is an absolutely insular state of affairs. And (3) yields that no proposition depicting (what would be) an absolutely insular state of affairs can be conceptually contingent. So it must be impossible, or necessary, that surdons exist. Since it is not impossible—since *surdon* is a "conceptually consistent concept"—the existence of surdons must accordingly be a conceptual necessity.

As Field remarks, this is not *per se* a consequence which Fregean platonism need find disquieting, since the platonist will hold that numbers exist of necessity and can allow, presumably, that they meet the definition of "surdon". What is meant to be disquieting is rather the method of delivery—as is attested by the fact, Field contends, that an exactly similar argument will deliver the necessary existence of *concrete* surdons, entities which ought to disturb even the platonist's ontological stomach.

Does this disclose a well-taken objection to (3)? Clearly the form of argument Field gives—though we grant that it has certain sound instances: cases where a true conclusion is validly derived from true premises—cannot in general be *suasive*: cannot be such as rationally to compel assent to its conclusion. To see that, merely reflect that the aura of conceptual consistency given off by the supposition that surdons exist is perfectly matched by the supposition that they do not. So once it is granted that (3) entails that the proposition that surdons exist is no contingency, we have only to elect to exploit instead the presumed conceptual consistency of the thought that surdons do *not* exist to conclude that the existence of surdons, so far from being a conceptual necessity, is actually *impossible*!

Someone might reply that that just intensifies the *reductio*—that it is merely grist to the Field mill if, more than delivering unwelcome results like the existence of concrete surdons, (3) can be made, in conjunction with merely commonsensical premises, to deliver contradictions. But that is not the right response to the situation. As a parallel, observe that it is, pre-reflectively, no less apparently conceptually consistent to suppose that a counter-example to Goldbach's Conjecture exists than to suppose that there is none—each supposition is readily intelligible and, so far as one can see, coherent. But here there is no analogue of principle (3) which might optionally be rejected—at least not if one accepts the ordinary view that Goldbach's Conjecture is no conceptual contingency but is either necessarily true or necessarily false. If we allow the transition from the apparent conceptual consistency of the Conjecture to its possible truth, we will therefore conclude it is necessarily true—a much easier proof than we had cause to expect, until we reflect that the transition from the apparent conceptual consistency of the supposition of counter-examples to *its* possible truth is at the service of an equally easy disproof. So there is a stand-off between the two impressions of consistency. Neither transition can be allowed until some further consolidating consideration—precisely a proper proof or refutation of the Conjecture—is adduced. Neither argument may be viewed as sound until such a consolidation is to hand. Neither argument is suasive in any case. The surdon argument is no better.

The evident lesson is that considerations of apparent conceptual consistency only *defeasibly* justify claims of possibility. To be sure, we—the present authors—are in no doubt about the conceptual consistency of *surdon*. But the seeming consistency of the definition of the concept is only a defeasible ground for that thesis. The crucial consolidating consideration is that since numbers, etc., conceived as by Fregean platonism, both exist and are surdons, the concept of a surdon is shown conceptually consistent by instantiation. But the *real ground* for the possibility claim is provided by the case for a Fregean platonist ontology. As a self-standing argument, the proof of the necessary existence of surdons is no better than the proof that their existence is impossible.

What, therefore, of Field's efforts to unsettle the Fregean platonist's digestion with concrete surdons? He writes:

> Perhaps one could make more of a problem for Wright and Hale by noting that the notion of a concrete (non-abstract) surdon also seems to the untutored mind to be perfectly *consistent* (however bizarre it may be to suppose it instantiated); so (3) would entail that even concrete surdons exist, and exist necessarily. Something, surely, has gone wrong. (1993, p. 297)

Well, indeed. Observe, first, that the point just made—that apparent conceptual consistency can only defeasibly justify a claim of possibility—is perfectly general: it is immaterial whether the type of entity whose existence is in question is abstract, or concrete. So the argument for the (necessary) existence of concrete surdons which Field seeks to foist upon us is certainly no better than the argument for the (necessary) existence of surdons *simpliciter*. Since the supposition that there exist *no* concrete surdons is certainly no less apparently conceptually consistent than the supposition of their existence, the former argument can be matched by an at least equally good argument for their necessary non-existence, just as in the latter case; so pending some consolidating argument on one side or the other, nothing more than a stand-off can be accomplished.

This consideration is by itself sufficient to neutralize Field's play with concrete surdons. However, it is in point to observe that the argument to concrete surdons is arguably in worse shape, just because the appearance of conceptual consistency in this case is anyway desperately thin. If being concrete involves, as it might well be thought to do, standing in causal relations with other things, the game is up right away; if not—if bare spatio-temporal extension, for example, is taken to be enough for concreteness—then an obvious dilemma looms: either surdons compete with other concrete objects for *Lebensraum*, inconsistently with their insularity, or they do not, in which case they would seem to be indistinguishable from bare spatio-temporal regions. Are the latter to be viewed as abstract or concrete? If abstract, then we have inconsistency once again. But if the right view—as Field himself might incline to think—is that bare spatio-temporal regions are concrete, then we have independent corroboration of the existence of concrete surdons, and hence of the consistency of the concept. So the argument to their existence from (3) may be viewed as harmlessly, if non-suasively, sound.[19]

Moral: the premise that there could be surdons, *a fortiori* concrete surdons, needs a better justification than Field gives it. It can get one, at least in the abstract case—but the mere apparent conceptual consistency of the concept is not, in this context, enough. One should indeed not expect platonism to be *that* cheap. Nor should one expect it to be so easy to discredit the presumption, articulated by (3), in favour of an integrated web of contingency in which no insularities occur.

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[19] To be quite fair, the point remains that—once we are satisfied of the consistency of the concept—the argument from (3) yields the *necessary* existence of concrete surdons. But if the ground for the consistency claim is that spatio-temporal regions actually are concrete surdons, then that is a problem only if the existence of spatio-temporal regions is properly conceived as a contingency. Let someone who thinks so make the case.

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