Nominalism and the Contingency of Abstract Objects

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NOMINALISM AND THE CONTINGENCY
OF ABSTRACT OBJECTS*

TRADITIONAL mathematical nominalism[1] regards a mathematical theory as acceptable only if it can be so interpreted as to involve no purported quantification over or singular reference to abstract objects. To whatever extent a theory fails this test, it is viewed as literally unintelligible. Hartry Field's[2] nominalism is atraditional in this respect. Field disbelieves in abstract objects, of course, and accepts that it is a constraint upon defensible mathematical thought that it avoid commitment to them. But he does not hold it to be a condition on the intelligibility of a mathematical theory that it allow of a non-Platonist *semantics*. On the contrary, he is prepared to grant that a Platonist semantics for, e.g., number theory is descriptively correct: that number-theoretic statements do indeed purport singular reference within and quantification over a specific domain of abstract objects. Since, in his view, there are no such objects, Field has to hold that such statements' truth conditions are systematically unrealized. And so, of course, he does.

How can anyone who holds this view—that math is massively false —hope to save the discipline without semantic reconstruction? By accepting a distinction, roughly parallel to that advocated by Bas van Fraassen[3] in the philosophy of science, between *endorsing* a theory

[1] See, e.g., W. V. Quine and Nelson Goodman's classic nominalist manifesto, "Steps towards a Constructive Nominalism," *Journal of Symbolic Logic* (1947): 105–22.
[2] As originally expounded in his *Science without Numbers* (New York: Blackwell, 1980).
[3] *The Scientific Image* (New York: Oxford, 1980).

and *accepting it as true*. For van Fraassen, one need not, in endorsing a physical theory, be committed to more than its empirical adequacy—its correctness as far as what is observable is concerned. And for Field, one need not, in endorsing a mathematical theory, be committed to more than its *conservativeness:* where a theory is, roughly, conservative with respect to a discourse just in case any inferences among statements of the discourse which it mediates could at least in principle validly be constructed—though perhaps at great cost in length and notational simplicity—without appeal to that theory.[4] For Field, a mathematical theory can be acceptable, even if its ingredient statements have Platonistic truth conditions, provided it is in this way conservative with respect to *nominalistic* discourse: discourse all of whose ingredient statements have truth conditions that make ontological demands only within the domain of the acceptably concrete. Field's program accordingly has the objective of showing that as much as possible of classical mathematics can be saved as acceptable by this standard.

A question arises, however, about the interpretation of the idea of conservativeness. No mathematical theory that employs a material conditional can be conservative unless consistent. But what understanding is one who disbelieves in abstract objects to have of the notion of consistency? Orthodox proof-theoretic and semantic accounts each involve quantification over nominalistically forbidden entities—*sequences* of sentences in the case of proof-theoretic accounts, and *models* in the case of semantic ones. Field ought, therefore, to forgo both of these approaches. And this is precisely what he does, construing consistency and, associatedly, the notion of consequence utilized in the concept of conservativeness, in *primitively modal* terms. Roughly: a statement is a consequence of others if and only if it is not *possible,* in a primitively modal sense, for it to be false while they are all true, and is inconsistent with others if and only if its negation is a consequence of them.

These proposals have a simple corollary. A consistent theory will be one whose axioms are *possibly collectively true,* where this possibility is to be understood in Field's primitive sense.[5] And a believer

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[4] In Field's own formulation: a mathematical theory S is *conservative* if, for any nominalistic assertion A and any body of such assertions N, A is not a consequence of $N + S$ unless A is a consequence of N alone. *Realism, Mathematics and Modality* (New York: Blackwell, 1989), p. 125.

[5] Reflect that a set of sentences—in particular, a mathematical theory—is consistent provided that each of its ingredients, S, is consistent with the remainder, $\{S_1, \ldots, S_n\}$. So if $\{S, S_1, \ldots, S_n\}$ is a consistent theory, then, following through on the foregoing construals, not-S will not be a consequence of $\{S_1, \ldots, S_n\}$, and hence it will be possible for $S_1, \ldots, S_n$ and not-not-S, $= S$, to be simultaneously true.

in the conservativeness of a particular mathematical theory is committed, on Field's account, to endorsing this possibility.

The objection that is to occupy us arises at this point from the consideration that Field's primitive operator of possibility is *object-linguistic*. It has to be, of course. The program would lose any advantage over more traditional versions of nominalism were it to construe claims about consistency metalinguistically. For if consistency is so construed, a nominalist who believes in the widespread conservativeness, and hence consistency, of mathematics is committed to a belief in nothing other than the feasibility of the traditional nominalist program—of providing concrete (or otherwise nominalistically acceptable) *models* of mathematical theories. And that is just what Field wanted to distance himself from. But when the operator is object-linguistic, Field's belief in the conservativeness of, e.g., number theory would appear to commit him to the view that the Peano axioms, say, might have been true as standardly interpreted, and hence that the nonexistence of numbers is a mere *contingency*. As a nominalist, he disbelieves in the existence of abstract objects. But number theory, he holds, is conservative, so consistent, so—if the reasoning just offered is sound—what its axioms collectively say might have been true. And if that had been so, there would have been numbers.

This may seem odd. But why is it an objection—why should the contingency of numbers and other abstract mathematical objects be awkward for Field? For at least two reasons. First, granted that standard mathematical theories are conservative, and that their deployment in the formulation of, e.g., physical theories can be avoided in the manner Field envisages, there is a question about the possibility of anyone having *grounds* for nominalism in the first place. If whether numbers exist is contingent, the question is presumably beyond a priori resolution. An appraisal will therefore demand empirical evidence. Field agrees with the present authors, however, in rejecting any idea of a *direct,* intuitional or "perceptual" epistemology of abstract objects.[6] And the very conservativeness of number theory, along with its dispensability in formulating scientific theories, would have the effect that there can be no nominalistically

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[6] Along lines, for example, famously suggested—or so it is often supposed—by Gödel (cf. "What is Cantor's Continuum Problem?" in Paul Benacerraf & Hilary Putnam, eds., *Philosophy of Mathematics: Selected Readings* (New York: Cambridge, 1983, 2nd ed.), pp. 483–4); or perhaps as developed by Penelope Maddy, "Perception and Mathematical Intuition," *Philosophical Review, LXXXIX* (1980): 163–96. It would take us to far afield to rehearse our objections to these views here. For some of them, see Wright, "Why Numbers Can Believably Be," *Revue Inernationale de Philosophie,* CLXVII (1988): 425–73, esp. pp. 464–5; and Hale, *Abstract Objects* (New York: Blackwell, 1987).

statable, *indirect* evidence for or against the existence of numbers. Whether or not the alleged contingency is realized would seem, therefore, to be in principle beyond evidence of any sort, and consequently utterly imponderable. So why is Field a nominalist? Why does he presume to a view on the matter?

In his recent book, reacting to this thought, Field writes as follows: "if the dispensability programme *can* be carried out"—that is, if it can be shown not merely that mathematics is conservative but that the hypothesis of mathematical objects is dispensable without loss in explanations, in descriptions of our observations, in accounts of metalogic, and so on:

> that gives us reason to not literally believe mathematics but only to adopt a fictionalist attitude towards it. Admittedly, we can't have *direct evidence* against mathematical entities. We also can't have direct evidence against the hypothesis that there are little green people living inside electrons and that are in principle undiscoverable by human beings; but it seems to me undue epistemological caution to maintain agnosticism rather than flat out disbelief about such an idle hypothesis. I think that platonism has seemed a plausible position because it has been assumed that the existence of mathematical entities is not an idle hypothesis. But if . . . the hypothesis is dispensable without loss . . . it is natural to go beyond agnosticism and assert that mathematical entities do not exist (*Realism, Mathematics and Modality*, pp. 44–5).

One observation about this response would be that the general form of transition it makes, from recognition of the necessary absence of all evidence germane to a hypothesis to a "natural" belief in its negation, is arguably completely unwarranted unless there is independent reason to think that there *would* be germane evidence if the hypothesis were true. That is precisely not the situation in the relevant case. But matters are worse than that. For idle hypotheses, in the sense Field means, have idle negations, too. So "flat out disbelief"—belief in the negation—of what is idle cannot in general be justified merely because it is idle. Agnosticism does not here reflect "undue epistemological caution"; it is demanded by mere consistency.[7]

There is a second and deeper reason, however, why the contingency of abstract mathematical objects might be thought an objectionable outcome. This is the reflection which the contingency in question would seem to be, metaphysically, absolutely *surd*—that there is no prospect of any account of what the alleged contingency

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[7] Or if there is a principle that sanctions disbelief in some such cases, it must involve reference to something more than idleness—and Field does not identify, still less support any such principle.

could be contingent on. It is deeply rooted in our ordinary concep-
tion of contingency that there ought in general to be explanations of
why things which might have been the case are not the case, and of
why things are the case which might not have been the case. But in
the present kind of instance—the existence of the integers, or real
numbers, for example—there is not even a glimmering of how such
an explanation might proceed. This seems a strong sign that the
notion of contingency would be *misapplied* in the present context,
and hence that there is philosophical error in any assumptions that
commit Field to so applying it.

Field has responded tersely to this criticism, charging that the idea
that he suffers any such commitment depends on an equivocation.
He writes

> Hale and Wright both note that I regard mathematics, and the exis-
> tence of mathematical entities, as consistent . . . and that I take consis-
> tency as a primitive modal notion, a sort of possibility. They then argue
> that since I regard it as false that there are mathematical entities, I
> must hold the existence of such entities to be 'contingently false'; and
> they both proceed to interpret 'contingently' in some non-logical sense
> (i.e. some sense other than 'neither logically true nor logically contra-
> dictory'), to make the position seem absurd. . . . It should be noted that
> if [their] objection were good, it would apply equally well against any
> platonist who is not a logicist: that is, any platonist who agreed that
> mathematics goes beyond mere logic, and hence that the denial of
> mathematics is logically consistent and hence 'contingent'. But
> of course it is not good, for as I've said it turns on an equivocation
> on the meaning of 'possible' (*Realism, Mathematics and Modality*,
> pp. 43–5).[8]

Field's response, then, is that the sense of 'possible' for which he
acknowledges commitment to the possible truth of the Peano ax-
ioms is not one that subserves the objection, which is consequently
defused.

But how exactly? There are two—and, so far as we can see, only
two—possible lines of thought that would be relevant.

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[8] The claim that "if [our] objection [i.e., the contingent falsehood objection]
were good, it would apply equally against any platonist who is not a logicist"
tacitly assumes, in effect, that no relevant notion of necessity (that is, no "abso-
lute" notion) is available, other than the austere notion Field describes. Obviously
enough, if it is *not*—save in that austere sense—a contingent matter whether
numbers exist or not, or whether or not number-theoretic statements are true
(i.e., if they are only *weakly* contingent), then the existence of numbers, and the
truth of number-theoretic statements, is, if not impossible (conceptually) neces-
sary. There is thus clear space for a version of Platonism, falling well short of
logicism as Field conceives it, that is not prey to the contingent falsehood objec-
tion we have urged against Field's own position.

(A) It might be maintained (i) that Field's preferred "austere" notion of logical contingency is the only one available; and (ii) that the objection needs a richer but dubious notion of contingency.

(B) It might be granted that there are available other, less austere but still respectable notions of contingency, but then maintained that while the nonexistence of numbers and other mathematical entities is contingent in the original austere sense, it is not a contingency—a fortiori not a metaphysically surd contingency—in the richer sort of sense demanded by the objection.

The considerations to follow will depend on no assumption about which, if either, of these lines might be Field's own preference.[9] Our interest is rather in the moves available in principle to the nominalism Field has advocated, irrespective of other aspects of its author's actual views. We shall argue that both strategies, (A) and (B), are fraught with difficulty. Since Field's claim—that the original objection is defused by reflecting that it is only to the "logical" contingency of, e.g., numbers that his belief in the conservativeness of number theory commits him—requires that one or the other strategy be defensible, it should be rejected as ill-considered.

The program is as follows. The next section will argue that claims A(i) and A(ii) are highly implausible at best. Section II will briefly explore certain problems with strategy B. This will lead naturally into a discussion, in section III, of certain internal tensions of Field's views about the semantics of mathematical theories with the sorts of consideration by which he seemingly regards Platonism as discredited.

Even if the original objection is not to be dismissed as relying on an equivocation, the nominalist still, of course, has the option of biting the bullet: of allowing that the existence of numbers and other abstract objects is indeed contingent in a richer sense than Field's austerely logical one and of attempting to maintain that the resulting position is unexceptionable. Section IV will attempt to deepen and refine the original objection in various respects. In particular, we shall begin to explore whether the nominalist has any prospect of a satisfactory head-on response to it via play with the idea of brute contingency.

I. STRATEGY A

Suppose that there were no other relevant notion of contingency than Field's austerely logical notion—according to which, approximately, a statement is contingent just in case it is neither (first-

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[9] Actually, Field has made it quite clear in conversations and correspondence that he would want nothing to do with the former.

order) logically true nor (first-order) logically contradictory.[10] How might someone proceed to argue—as per A(ii)—that the contingency objection would then be stifled?

One idea might be that, if we introduce 'it is logically contingent that' as a sentential operator, simply and solely as a definitional abbreviation for 'it is neither first-order logically true nor first-order logically contradictory that', we have—so far, anyway—done nothing to provide a sense for statements of the form 'the fact that . . . is contingent upon the fact(s) that . . .', and so we cannot so much as intelligibly formulate the question, "What is the (allegedly) contingent fact (that there are no numbers) contingent *upon*?" Nothing in the way in which we would have introduced the contingency operator would prepare for the idea that when it is (merely) contingent that the things are thus and so, there is—still less has to be—something else upon which their being thus and so *depends*.

This, however, has no force. Asking what the putative contingency, that there are no numbers, is contingent on is a convenient and natural way to raise the kind of question that fuels the objection. But we can just as well ask *why* there are no numbers,[11] given that it is, in Field's austere sense, contingent that there are none. That is a perfectly well-formed and legitimate request for an explanation, for which no further preparation, supplementary to Field's account of contingency, is needed. According to Field, it is logically contingent but false that there are numbers. Well, there are many statements that are logically contingent but false, and it is typically

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[10] In fact, this oversimplifies Field's notion. Field's explanation (*Realism, Mathematics and Modality*, pp. 32–8) of "LTrue" (read 'it is logically true that'), in terms of the laws governing it, makes it clear that many statements involving this operator would rank as (austerely) logically true, and that various statements involving second-order quantification would be among them. It is thus strictly incorrect to suggest that a statement is logically contingent for Field if it is neither (first-order) logically true not (first-order) logically contradictory. The fact remains that (austere) logical truth, contingency, and impossibility are, for Field, essentially a matter of logical form—the point is merely that this is not exhaustively characterizable in first-order terms. It should be clear that none of our arguments makes essential play with this simplifying assumption adopted here—in particular, there is no question but that the examples of (austere) logical contingency discussed in this section would rank as such, even given Field's own slightly more generous characterization of the notion.

[11] As indeed, in one of the passages to which Field is responding, Wright in effect does. The passage, which Field actually quotes (cf. *Realism, Mathematics and Modality*, p. 43), runs: "Field has no prospect of an account of what the alleged contingency is contingent *on*. The world does not, in Field's view, but might have contained numbers. But there is no explanation of *why* it contains no numbers; and if it had contained numbers, there would have been no explanation of that either. There are no conditions favorable for the emergence of numbers, and no conditions which prevent their emergence" (Wright, *op. cit.*, p. 465).

perfectly proper to ask *why* these statements are false. So a special consideration would be needed—and none is so far in sight—to justify rejecting the question as improper in the present case.

Admittedly, there is a gap between its being "typically perfectly proper" to ask why an austerely contingent matter is as it is and there being, on the other, an *internal conceptual connection* ensuring the availability of at least an appropriate *category* of explanation for each austerely contingent matter. We make no claim to have closed this gap, and it is thus theoretically open to a supporter of Field to attempt to exploit it—more about that later. The fact remains that asking for an explanation of why an austerely logical contingency is unrealized is, at least in a wide class of cases, perfectly legitimate. If there are other austere contingencies where the demand for explanation is out of order, there ought to be some stateable distinction between the two sorts of cases. And argument will be needed that the existence of numbers belongs in the latter class. There is no progress otherwise.

It is in any case very implausible that, in line with A(i), a restriction to austerely logical contingency could defensibly be maintained. Any statement is austerely logically contingent whose most explicit first-order formalization is neither a theorem of first-order logic nor refutable therein.[12] So, for instance, both

(i) There are vixens weighing in excess of eighty pounds.

and

(ii) There are male vixens.

rank as austerely logical contingencies. More generally, austerely logical contingency will embrace many statements whose truth it is attractive to regard as, in old-fashioned terminology, *conceptually* impossible but a first-order demonstration of whose impossibility, if one could be given at all, would require supplementary content-explicating postulates of a nonlogical sort. Now, in each of the kinds of example typified by (i) and (ii), respectively, the request for an explanation of *why* the statement in question is false is apparently perfectly intelligible, and it is easy to say how at least the beginnings of an answer should proceed. Given that first-order logic does not preclude the truth of such a statement, the question is, what does? And the answers are, roughly, the typical genetic make-up and feeding habits of vixens, and the nature of the concept of a vixen, respectively.

[12] But see note 9.

So much is merely common sense. But the question now is: How should a proponent of A(i) respond to the demand for an explanation of why there are no male vixens? How to respond without betraying an understanding of a broader notion of contingency than the austerely logical variety—a notion of contingency that, if numbers are so contingent, will subserve the original objection, even if, contrary to what was just suggested, austere logical contingency will not?

Well, reflect on the possibilities. In effect, the question—"Why are there no male vixens?"—sets a trilemma:

(i) There is the option of rejecting the demand for an explanation as illegitimate.
(ii) There is the option of attempting an explanation similar to that of why there are no vixens weighing in excess of eighty pounds.
(iii) There is the option of attempting an explanation that preserves a distinction between the two kinds of example.

Option (i) is surely hopeless. It simply is not credible to suppose that there is nothing at all to be said about why there are no male vixens.[13] But it would be equally incredible to suppose that a proper explanation might assimilate the two examples, attributing the non-occurrence of male vixens to *causal* factors, specifically, to the genetic make-up and typical environmental conditions of foxes. Someone who thought that a satisfactory explanation might proceed along those lines would owe a story about what kind of variation in those factors might secure the emergence of male vixens. But there is no such story to tell.

The fact is that only option (iii) is a credible response. More specifically, however indigestible the thought and unpalatable its terminology may be to philosophers weaned on the thin nutrient of W. V. Quine's "Two Dogmas of Empiricism,"[14] the only credible response is to acknowledge that it is not in the nature of the concepts *male* and *vixen* to allow coinstantiation. There may or may not be more to say in response to the request for explanation in this particular case—but that much must be at least the basis of any satisfactory response.

As soon as it is granted, however, that there is work to be done by notions of conceptual possibility and impossibility which apply differentially within the class of austerely logical contingencies, houseroom has been given to just the kind of richer notion of contingency

[13] A parent who so responded to her child's question would surely be failing to explain something that a responsible parent ought to want to explain.
[14] In *From a Logical Point of View* (Cambridge: Harvard, 1980, rev. ed.).

that it was essential to strategy A to repudiate. Say that a statement is *strongly contingent*, just in case it is both austerely logically contingent and not conceptually impossible: no impediment to its truth is generated either by first-order logic or by anything internal to the character of the nonlogical concepts that it involves. If this notion is granted and, with it, the resources for a distinction, among austerely logical contingencies, between those like "There are vixens weighing in excess of eighty pounds" which are also strongly contingent and those like "There are male vixens" which are not, the objection immediately resurges in the form of a dilemma, namely, to which camp should "There are natural numbers" be assigned?

If to the latter—if the existence of numbers is regarded not as a strong contingency but as a conceptual impossibility—then that is to abandon strategy A in favor of strategy B. But if to the former—if the existence of numbers is conceived as a strong contingency—that immediately invites the assimilation of "There are natural numbers" to "There are vixens weighing in excess of eighty pounds"—a statement whose falsehood allows of a broadly *causal* explanation. And the idea that such an explanation might be possible of why there are no numbers is not merely in tension with our ordinary conception of abstract objects as acausal but is demonstrably inconsistent with the belief that number theory is conservative with respect to nominalistic discourse.[15]

Even if the presumption of *causal* explicability can be defeated, there is no way round the point that to conceive of the existence of numbers as a strong contingency is a commitment to the view that nothing in a correct understanding of number theory or its underlying logic poses any obstacle to its truth. The idea would have to be, rather, that *it is merely a feature of nature* that, if it is, the world is numberless. And to try to take that conception of the matter seriously is simply to point one's chin at the original question: *Why* has nature turned out that way—what kind of consideration might in principle explain the nonexistence of numbers once they are acknowledged to be a strongly contingently nonexisting class of enti-

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[15] *Proof.* Suppose that a causal explanation of the nonexistence of numbers could be given. Then assuming that such an explanation will be of the familiar deductive-nomological pattern, there will be some nominalistically formulated laws (briefly, L) and nominalistic statement of initial or standing conditions (briefly, A) from whose conjunction the statement that there are no numbers could be derived. We may assume that the conjunctive premise is consistent, and that neither conjunct alone entails this conclusion. Let N be elementary number theory. Then, since N entails that there are numbers, not-A is a nominalistic statement entailed by $N + L$ but not by L alone, whence—bearing in mind Field's definition (cf. note 4 above)—N is not conservative.

ties? And how can it be legitimate to think of the matter as, so to speak, merely natural, unless that question has some kind of constructive answer?

II. STRATEGY B

The question can be avoided by reverting to strategy B. This, recall, involves granting that there is, in addition to any austerely logical sense of contingency like Field's, a richer notion; and then claiming that while the existence or nonexistence of numbers is a contingent matter in the former sense, it is *not* so in the richer sense. The question—"Why are there no numbers?"—would thus admit of a direct answer: the nominalist could simply adduce whatever considerations show that numbers are impossible in the richer sense.

The problem, obviously, is to locate a suitable richer notion—one that allows successful argument that numbers and other abstracta are indeed richly impossible. The simplest line, naturally, would be to attempt to maintain that the existence of numbers is conceptually impossible in the relatively straightforward manner of male vixens, or squares with unequal sides. On this view, all existentially committed classical arithmetical theorems would conceal definitionally elicitable inconsistency. One difficulty with this is that it is crucial to Field's project that arithmetic be formally consistent (else it cannot be conservative). And presumably its formal consistency should be sufficiently robust not to be disrupted merely by expansion of its statements in the light of correct definitions of certain of the concepts involved. But there seems, in any event, no prospect of making the necessary case, i.e., of a successful argument that while there is no overt formal inconsistency in, e.g., the (normally regarded as true) statement that $117 < 5$, it can nevertheless, with the help of acceptable definitions of the specifically arithmetical terms involved, be made to give way to explicitly contradictory consequences, in the manner in which the statement that there are squares with unequal sides gives way to an explicit contradiction on replacement of the geometrical term 'square' by its normal definiens.

If there is to be a conceptual impossibility here, it has to be of a subtler sort. Actually, it is hard to see how the alleged impossibility could be held to derive from anything but the very abstractness of numbers, so that the existence of any kinds of abstract object would have to be impossible. Of course, taken in one way, this would merely inherit the difficulties just glossed. For it seems equally implausible to suppose that the very concept of *abstract object* is self-contradictory after the fashion of *male vixen* or *four-sided triangle*. But perhaps it could be argued that the concept, though not

self-contradictory, is nevertheless defective in such a way as to guar-
antee that it can have no instances.

How? Field's writings endorse *passim* a number of familiar anti-
Platonist lines.[16] In particular, he contends—in company with many
others, of course—that the acausality and lack of spatiotemporal
position of Platonistically conceived mathematical entities make for
serious, if not insuperable, difficulties over knowledge of and refer-
ence to them. He also endorses the well-known arguments against
Platonism given by Paul Benacerraf[17] and by Hilary Putnam.[18] Un-
less bolstered with some substantial further premise, however, none
of these arguments will deliver the type of conclusion that strategy B
requires, namely, that there can be no such objects as numbers, sets,
etc. For instance, the proper conclusion of a cogent argument based
upon some presumed causal constraint on knowledge would be
merely that, *if* there are such objects as numbers, we can have no
knowledge concerning them. Likewise, the upshot of the arguments
—whether based upon some proposed causal theory of reference,
or upon the kinds of considerations adduced by Benacerraf or Put-
nam—purporting to establish that (determinate) reference to num-
bers is impossible, is just *that*, i.e., that, *if* there are such things as
numbers, there is no way we can make (determinate) reference to
them. If these arguments are to yield the further conclusion that
there simply cannot be any such objects at all, then further premises
are needed, perhaps directly to the effect that no objects can exist of
which knowledge is in principle impossible or to which there cannot,
even in principle, be identifying reference, or perhaps yielding the
desired result some other way. In any case, it is clear that proving
the impossibility of numbers, or abstract objects generally, is a very
tall order which no extant line of philosophical argument meets. If
nominalists had such an argument, we would have heard about it.

But if the case were somehow made, there would be a further
issue, worth briefly airing. Field, remember, has no quarrel with
Platonism as an account of our actual understanding of mathemati-
cal language. And this literalism is an attractive and essential ele-
ment in his kind of nominalism, distinguishing it from nominalist
programs of the more traditional reconstructive (and failed) sort.
What, in his view, is wrong with Platonism is not that it misrepre-
sents our actual understanding of mathematical statements, or

[16] *Realism, Mathematics and Modality,* pp. 68–70, also pp. 23 and 230ff.
[17] "What Numbers Could Not Be," *Philosophical Review,* LXXIV (1965): 47–
73.
[18] "Models and Reality," *Journal of Symbolic Logic,* XLV (1980): 464–82.

credits them with a metaphysically impossible content, but merely that it tells ontological untruths. But suppose we had the impossibility proof that strategy B demands. What repercussions ought such a discovery, once it were generally known, to have on the semantics of mathematical statements? Could it be appropriate to continue to take the literalist view?

It may seem that exactly this option—combining a literal construal with denying the possible existence of mathematical entities —is what is secured by going *fictionalist*. No doubt it would be absurd to represent a community as *sincerely* making mathematical claims referring to entities that they regarded as impossible. But a face-value construal of their mathematical language need not involve doing so, provided we can avoid interpreting the commitments that their mathematical assertions express as *beliefs in the truth* of those sentences. And that is just what mathematical fictionalism enables us to do. Fictionalism opens a gap between (i) acknowledging that, if the belief prevails in a community that there are, necessarily, no abstract objects, it cannot be part of the best semantic account of the community's mathematical assertions to represent their *makers* as purporting reference to and quantification over such objects, and (ii) abandoning the face-value construal of the terms and quantifiers that their *mathematical language* contains. The gap is opened precisely because fictionalism makes a distinction between endorsing a mathematical statement and accepting it as true.

There is a residual concern, however. Only fictionalism, it seems, can reconcile three crucial ingredients in Field's philosophy of mathematics: (i) a face-value construal of the semantics of number-theoretic language, (ii) the—in his view—justified disbelief that any actual objects answer to the terms and quantifiers discerned therein, and (iii) our endorsement of the axioms and theorems of number theory. But the envisaged case, remember, is one in which we are supposed to be rightly persuaded not merely that there are not but that there *could not* be numbers—the fiction has to be one of acknowledged *impossibilia*. There will therefore be issues about the *intelligibility* of mathematical language, construed at face-value, which fictionalism as such does nothing to address.

The interactions between intelligibility and the perception of various forms of conceptual impossibility are of course subtle. One might believe in the incoherence of the idea of time travel, for instance, yet still, to all intents and purposes, understand and enjoy a fiction in which such technology plays a central part. By contrast, many of us are persuaded of deep conceptual singularities in the

idea of the *Cartesian ego* as a temporal continuant—persuaded, e.g., that there is no coherent notion of what it would be for such an entity to endure through a given period of time. Would there nevertheless be no difficulty about devising an intelligible fiction in which such entities were to be the central—repeatedly referred to and quantified over—characters? Again, assuming—plausibly—that there are conceptual obstacles in the way of interpreting an imagined community as genuinely operating with a deviant arithmetic, there is a real question concerning the intelligibility and coherence of a fiction in which the protagonists practice an arithmetic in which $7 + 5 = 13$, etc.[19]

Obviously, in the absence of any clear indication of the kind of conceptual impossibility that a successful prosecution of strategy B might disclose, or the kind of considerations that might be involved, there has to be an element of shadow boxing about this. Nevertheless, there are, unquestionably, kinds of conceptual incoherence that would jeopardize the intelligibility even of fictional reference and quantification. So there can be no a priori assurance that an impossibility "of a more subtle sort" which pursuit of strategy B might disclose will not have such an effect—no assurance that the strategy could be prosecuted without casualty to the semantical literalism that distinguishes Field's position from nominalism of more traditional kinds.

III. LITERALISM AND THE STANDARD ANTI-PLATONIST ARGUMENTS

That there is cause to doubt the coherence of literalism with other aspects of Field's views may be brought out by recalling some of the considerations to which Field actually appeals as discrediting Platonism—in particular, the allegedly irremediable indeterminancy disclosed by Benacerraf's argument and the idea that their acausality and lack of spatial position preclude any satisfactory account of how reference to numbers is possible. For these considerations, if cogent at all, seem by themselves quite sufficient to make trouble for a face-value construal of, e.g., number theory, even in the context of an overall mathematical fictionalism. If refer*ence* to numbers is ruled out on conceptual grounds—whether or not the further step is taken of concluding that the existence of the refer*ents* themselves is impossible—it is immediately cast into doubt whether the face-value construal adds up to a *coherent* semantic story, let alone a preferable one. Nominalism-cum-literalism has it that arithmetical

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[19] Of course, we can readily make sense of a fiction in which the protagonists are depicted as making, more or less systematically, what would, by their own lights, be reckoned arithmetical mistakes; but that is beside the point.

statements, as ordinarily understood, have determinate though unrealized (or only vacuously realized) Platonistic truth conditions. But in order for any statement to have determinate truth conditions, there must be something correct, ergo coherent, to say about the reference of its singular terms, the satisfaction conditions of its predicates, the range of its quantifiers, and so on—and no *coherent* account of these matters, in the nature of the case, can stand in violation of general conditions on the very possession of such semantical characteristics. A face-value construal of mathematical statements—even if offered in a fictionalist spirit—is bound to offend in this respect if it involves attempting to assign reference to the relevant terms in a way that clashes with deep intrinsic features of the reference relation itself; and just that, according to proponents of the Benacerraf and acausality arguments, is what it must involve if it represents those terms as purporting reference to *abstract* objects.

To put the matter another way: one simply has not given a semantics for a discourse if the details of the proposed account controvert essential features of the semantical notions that provide its very framework. But, according to the arguments in question, to represent, e.g., the numerals as having reference, if to anything, then to abstract objects, is precisely so to controvert essential features of the relation of singular reference. Somebody who accepts such arguments should therefore hold that, rather than deliver unrealized (perhaps necessarily unrealizable) truth conditions for the sentences of arithmetic, the face-value construal gives implicit offense against the semantic relations on which it needs to rely in order to say anything determinate about what the truth conditions of arithmetical sentences are—and thus fails to deliver anything at all.

If this is right, then any anti-Platonist driven by the familiar arguments cited approvingly by Field is left, so far as we can see, with just three options. The first is *traditional nominalism,* according to which the appearance of singular reference and quantification presented by mathematical discourse may be genuine, provided the objects referred to and quantified over are concrete. The second is a *reductive* program to disclose that appearance as spurious, and eliminate it. The third is to accommodate the appearance in the only other way that seems open—to go *formalist* and reject altogether the idea that mathematical statements are bona fide contentful statements having genuine truth conditions. On the latter view, such statements possess a determinate syntax, which may be exploited in formal derivations, but no semantics—a fortiori, not a Platonistic one.

What is not apparently an option, for anyone driven by the familiar arguments, is Field's own view, or any view that shares the idea that mathematics, while a fiction, is a *Platonistic* fiction.[20]

IV. BRUTE CONTINGENCIES

Let us see where we have arrived. The literalist can avoid Platonism only at the cost of denying that mathematical theories are true. The question is then unavoidable: Do such theories fail of truth—specifically, do mathematical singular terms fail of reference—necessarily or merely contingently? The preceding suggests that the former option, in effect that pure mathematics is a fiction of impossibilia, is not the way for a supporter of Field to go. It remains to reconsider the alternative, that mathematical and abstract objects generally exist or fail to exist by grace of strong though inexplicable contingency—precisely the position targeted by the main objection.

The objection could be neutralized if it could be shown that the demand for explanation is here illegitimate; that there are, in any case, certain strongly contingent matters—matters to which the case of mathematical objects may properly be compared—of which, however they are resolved, there is no conceivable explanation. "No conceivable explanation" because not every contingency need, of course, be explicable *in fact*. For instance, if only subsumption under ulterior physical laws can explain the obtaining of any particular physical law, then it will be vain to seek an explanation of any *fundamental* physical law, if such there be. Still, there will be *conceptual* space—as it happens unoccupied—for an explanation of such a law; (for the law might not have been fundamental). By contrast, someone who wants to dismiss the question—"Why are there no numbers?"—as inviting an explanation of a matter that is actually a brute contingency, will need to meet the point that there seems to be no corresponding conceptual space—we seem to have no conception of how an answer might run in principle.

Pursuing this direction will involve a two-stage operation: first, a search for other relevant examples of brute contingency—contingencies where we conceive not merely that no explanation need be possible of the direction in which it is resolved but, more, that there simply is no *category* of explanation that might in principle serve; and second, an argument that the existence of numbers, and of abstract objects generally, belongs with this group. What are the prospects?

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[20] Doubts along the lines of the last few paragraphs are aired by Hale in "Nominalism," in A. D. Irvine, ed., *Physicalism in Mathematics* (Cambridge: Kluwer, 1990), pp. 136–7.

In correspondence, Field has remarked:

> On the question of why there are no numbers, my view is that this is
> like why is there matter, or why are there no immaterial minds, or why
> is there no God: you simply can't expect there to be an answer to such
> basic existence and non-existence questions ('basic' in the sense of:
> applied to a whole 'ontological category').[21]

This is a very natural thought, and it is therefore important to see
that it comes to not very much. For unless the claim is to be simply
dogmatic, it needs the backing of an *account* of "basic ontological
category" which both discloses why explanations of "basic" matters
of existence and nonexistence are indeed not to be expected and
also ensures that numbers, and abstract objects generally, turn out
to be a basic ontological category in the relevant sense. But it is not
clear how such an account might run. Indeed, it is obscure what
intuitive conception of a basic category covers each of the cited
examples. Notice that certain quite attractive proposals will not sub-
serve Field's thought in any case. Persons, and living organisms, will
both be basic on many accounts, including—though its author ex-
presses some uncertainty—the largest-class-of-objects-sharing-crite-
ria-of-identity idea of Michael Dummett.[22] But presumably it is not
absurd to seek explanations both of why there are any persons and
why there is life.

There are, however, other cases which may be held to exhibit
brute contingency, and so to provide the model Field needs. Con-
sider the ageless conundrum—"Why is there something, rather
than nothing?" An unanswerable question if physical existence is at
issue and a satisfactory explanation why a physical state of affairs
obtains has to advert to a causally prior situation in which it does not
obtain. For a physically empty world would presumably have no
causal progeny. Or consider the "great contingency"—the global
state of affairs that verifies all actually true contingent proposi-
tions.[23] Since the obtaining of each of its ingredients is a matter of
contingency, the exact character of the global state is no doubt itself
a contingency. But there can be no explaining its exact character—
at least not if contingencies can be explained only by reference to
further contingencies. For there are, precisely, no *further* contin-
gencies to appeal to; everything further is a matter of necessity.

[21] Letter of September 8, 1990.
[22] *Frege: Philosophy of Language* (London: Duckworth, 1973), pp. 75–6.
[23] Here we are indebted to Peter Carruthers and Gideon Rosen for suggestions
and helpful discussion.

Do such examples offer any comfort to a supporter of Field? To stress: it will not do just to observe that a certain sort of brute contingency is illustrated by examples of this sort, and is therefore at the call of the nominalist according to convenience. The crux will be whether "There are no numbers," or "There are no abstract objects," comes in the same rather special sort of category. And argument is required. The original objection was that the concept of contingency has no proper application to the cases in question, on the ground that contingencies invite the question—"Why?"—and that we have, in the cases in question, no concept of the form a satisfactory answer might assume. The onus is therefore on the nominalist to show that this is a bad reason—that the correct account of our lack of any such concept is not that the idea of contingency is being stretched beyond the limits of its intelligible application, but that—as in the case of the two mooted examples—explanation is here pre-empted in perfectly intelligible ways, quite compatibly with the claim of contingency.

Reflect on what would be demanded by a serious analogy with those two examples. There the essential work is done by certain restrictive principles: roughly,

> The explanation of the obtaining of a (physical) state of affairs must advert to a causally prior state of affairs in which it does not obtain.

and

> The explanation of a contingency must proceed in terms of further contingencies.

The key point is that such restrictions pre-empt explanation as soon as something sufficiently general is put up as the explanandum, and that this—rather than any misappropriation of the notion of contingency—is at the root of our inability to see how a satisfactory explanation of the contingencies in question might proceed. Accordingly, to establish a strict parallel, the nominalist needs to call attention to a restriction on explanations of mathematical existence which has a corresponding effect: our inability to explain the existence, or nonexistence, of numbers, or abstract objects generally, should be traced to the fact that the demand to do so implicitly transgresses bounds within which a satisfactory explanation would have to work.

The concept of mathematical explanation is a difficult one. But we can at least take it that explaining pure mathematical facts is always a matter of giving proofs, even if not all proofs are explanatory. That might suggest a suitable line for the nominalist. Mathe-

matical proofs, it might be argued, accomplish demonstrations, and therefore explanations, only *within a framework* supplied by the basic assumptions, including assumptions of existence, made by the relevant mathematical theories. Mathematical why-questions, the suggestion continues, are *internal* questions: grant the framework and ontology presupposed by the Peano axioms, and we can explain the infinity of the primes; grant the framework and ontology of Zermelo-Fraenkel set theory and we can—or so it is sometimes thought—explain the truth of the Peano axioms. But just as in the ascent to "Why is there any physical substance at all?" or "Why is the totality of contingencies composed just the way it is?" escalating generality of this sort must sooner or later clash with the possibility of explanation. It has done so by the time we ask why there are (no) mathematical objects.

Although this may seem to pass muster, on closer inspection the analogy limps at the crucial point. What exactly is the restrictive principle in play? Presumably, something like,

> The explanation of any matter of mathematical existence or nonexistence can only be accomplished within a framework in which the existence of a range of mathematical objects is assumed.

But that principle overlooks an asymmetry in the character of mathematical explanations of existence and nonexistence, respectively. No doubt it is only if the existence of the natural numbers is granted that someone can set about trying to explain why the ratio of primes $\leq n$ becomes smaller and smaller as n increases. But what about the explanation offered by Euclid's proof of the *non*existence of a greatest prime? Here it is inessential to the success of the explanation to presuppose that there really are numbers. For the proof succeeds in any case: if the natural numbers exist, then, by Euclid's reasoning, it is in their nature that there is no finite limit to the occurrence of primes; and if they do not, then there are no prime numbers and, a fortiori, no greatest among them.

The point is perfectly general. Mathematical explanations of nonexistence proceed, without existential presupposition, by showing that the corresponding existence claim clashes with essential aspects of our *concept* of the type of mathematical objects concerned, that the concept's being fully and properly exemplified would be inconsistent with the truth of the existence claim. There is no range of objects demanded by all such explanations, whose existence or nonexistence is thereby pushed beyond explanation. The candidate restrictive principle is false.

No firm conclusion can be drawn, of course, on the basis of this one failed comparison. But is there a better one? The heart of the objection is the claim that, while there is no simple connection between contingency and the possibility of explanation, there is nevertheless a *presumptive* connection: explicability is the default status, and the exceptions have to be *explicable* exceptions—something has to be done to show *why* ordinary explanation is precluded. The implicit principle, *P*, is that, for any putative (strong) contingency *C*, it should be possible *either*

(a) to indicate at least a category of explanation that an explanation of *C* might in principle instantiate—to gesture at the lines along which an explanation of *C* might run, even if as a matter of fact no correct explanation of that kind can be given; *or*

(b) to explain why there cannot be any such category—why any explanation of *C* would be required, by virtue of certain special features of the case, to meet unsatisfiable conditions.

And the backing for the principle is merely the natural idea that since, when it is a (strong) contingency that *C*, things might have been otherwise, it ought to be perfectly appropriate to ask *why* they are not otherwise, i.e., why *C*. Since this question is appropriate only if a suitable category of explanation exists, the onus has to be on someone who accepts that no such category exists but who is determined that *C* is nevertheless a (strong) contingency, to show why, in this special case, all space for explanation is closed off, i.e., to satisfy the second disjunct of the principle. This option can be nicely accomplished in the case of the two examples reviewed. But we have so far no clear suggestion how it might be accomplished for the putative contingency that there are no numbers.[24]

All this, however, is undeniably inconclusive. Can we conjure a wider class of examples that may help to place the putative contingency of mathematical objects in a more luminous perspective? We already have

(1) The "great contingency";
(2) "Why is there anything material?"—the Conundrum;
(3) Any fundamental scientific law, *L*;
(4) The existence of God—or, presumably, of any Being whose concept, like that of the Christian God, requires that its existence depend on nothing else.

[24] It is important to remember, moreover, that, since we are at this point concerned with a theorist who regards the strong contingency of numbers as the correct account of their modal status, hence as something imposed by an ordinary understanding of number—it has to be possible to draw on *agreed aspects* of that understanding to discharge the obligations of course (b).

Other examples do occur. Suppose physics suggests, for instance, that the universe is approximately n years old. Then the question

(5) Why did the universe begin approximately n years ago?

will admit of no causal explanatory answer since it presupposes that, prior to that period, there was no physical process—hence nothing to determine that process should start then rather than earlier. Or consider an experiment in which under radioactive decomposition a particle escapes through one slit in a screen rather than another, and suppose that—so we conceive—activity at this micromaterial level is subject only to statistical—indeterministic—law. Then there need be no answer to the question

(6) Why did the particle pass through the top slit?

No doubt such examples put paid to any lingering attraction in the simple thought that, for any bona fide contingency, there has to be something *on* which it is contingent. But collectively they merely reinforce the presumptive connection asserted by the principle, P. For in each case we can either indicate quite definite lines along which explanation might have been possible—L might have been subsumable under ulterior physical law, for instance, or the laws governing the relevant level of micromaterial activity might have been deterministic—or else, as in cases (1), (2), (4), and (5), we can point to clear conceptual considerations which, drawing just on our ordinary understanding of the example, preclude explanation in principle.

But there *are* more awkward customers. Consider any contingency, C, for which, as in cases (3) and (6), we possess a general conception of how an explanation might in principle proceed but where no correct explanation of that sort can actually be given. Then the circumstance—call it C^*—that there is no explanation of C will itself be a contingency, and it will be liable to be brute. For how might an explanation of C^* go? Not a priori, presumably, since C^* is a contingency—there might have been an explanation of C. But what sort of *empirical* explanation might be given of the contingent absence of all empirical explanation of a given (as it happens fundamental) law, or of the (as it happens) indeterministic character of some range of micromaterial events? Thus, for any C of the type illustrated by (3) and (6), the corresponding contingency of the form of C^* already poses a problem to the taxonomy implicit in P—it will be, prima facie, a case where it is clear neither what a suitable category of explanation might be nor that there can be, a priori, no such category. Thus we can add to the list, for any fundamental law L,

(7) There is no explanation of L's obtaining.

and likewise, in respect of example (6),

(8) There is no explanation of the particle's passing through the top slit.

More generally, for any of a wide range of physical phenomena which we can seemingly readily conceive might have been otherwise and yet of which our best current conception is that they have no physical preconditions—the whole phenomenon of gravitational attraction, for instance, or space's being non-Euclidean, or n-dimensional; in all such cases, the claim that there is no explanation of the phenomenon will seemingly be itself a contingency about which we are clear neither along what general lines an explanation might be given nor about what if any conceptual considerations preclude explanation in principle.

Well, are not these reflections enough to dislodge principle P? Do we not have to admit now that the intuition of contingency can perfectly justifiably be robust in circumstances where we are unclear whether either limb of principle P can be satisfactorily made out? Of course, these, as it were, second-order contingencies are of a special sort, and bear no immediate analogy to the putative contingency that there are no abstract objects. Still, the principle that drives the objection is not, it appears, exceptionless in any case. So what cause for skepticism if the nominalist postulates a further class of exceptions?

In an earlier discussion Wright wrote:

> . . . the idea of a contingent state of affairs that is contingent on nothing, admits of no evidence even in principle, and is beyond anything we could justifiably regard as cognition, is simply not a credible piece of metaphysics. . . .[25]

Focusing, as we have been, on the question of explanation belongs with the thought that bona fide contingencies are presumptively contingent on something, and that the exceptions should be few and clearly explicable as such. But we should not lose sight of the second component in the quoted complaint—the utter epistemic isolation of the existence or nonexistence of mathematical objects, when construed as a (strong) contingency. Moreover, it is important to be clear that the offense given thereby need not depend on any implicit

[25] Wright, "Why Numbers Can Believably Be," p. 465; cf. also Hale, *Abstract Objects*, pp. 111–2.

verificationism—any general aversion to the idea of evidence-transcendent fact. The reason there can be no cognition of, nor evidence for or against, the existence of mathematical objects, conceived as a strong contingency, is because *nothing whatever*—in particular, no uncontroversial contingency—varies as a function of their existence or nonexistence, save precisely that.[26] It is merely one consequence of this that their existence or nonexistence exerts no differential influence on what is cognitively possible. The conception that mathematical objects exist or not as a matter of strong contingency thus clashes not merely with the presumption that contingencies should be contingent on something but, equally, with the converse principle that bona fide contingencies are presumptively *things on which other bona fide contingencies are contingent.*

There is, of course, a vast range of ordinary unproblematic contingencies that respect both these conditions. That is the normal case. But what is striking is that *all* the examples we have considered, even those which prima facie make trouble for P, respect the converse principle. There are manifold uncontroversial contingencies that would not be so but for the detail of the "great contingency," the existence of matter, the obtaining of a particular scientific law, the universe's being n years old, space's being n-dimensional, the particle's going through the top slit (it leaves traces), God's existing (if he does), L's being fundamental (for if L were not fundamental, other superior laws would obtain and would be manifest in differential behavior of matter), and there being no explanation why the particle went through the top slit (for if there were an explanation, the relevant class of phenomena would not be indeterministic and all sorts of things would be different).

The suggestion, then, is that both the following presumptive principles constrain the ascription of strong contingency:

(1) That a putative contingency will be contingent on something— there will be circumstances on which its obtaining depends (and whose citation can thus contribute to its explanation).
(2) That a putative contingency will have things contingent on it— there will be circumstances whose obtaining depends on it (would be otherwise if it did not obtain.)

Brute contingencies are counterexamples to the first. Call a contingency *barren*, by contrast, if it is a counterexample to the second. Then none of the brute contingencies reviewed in our list of eight

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[26] If this were not so, then mathematical theories would not be conservative with respect to nominalistically stateable consequences.

seems to be barren. Uncontroversial examples of barren contingencies are, indeed, hard to muster. The occurrence of a mental state or episode, viewed in the special way characteristic of epiphenomenalism, would be one possible type of example. But even that would not also, of course, be brute. What is striking about "There are (no) numbers," viewed as a strong contingency, is that, at least with respect to all other uncontroversially contingent matters, the contingency it putatively expresses is absolutely *insular*—is both barren *and* brute. We conjecture that there is no other example of a contingency—at least, no case that would pass without controversy as a contingency—that has both those features.

It would be satisfying to disclose some definite metaphysical incoherence in the notion that there can be absolutely insular contingencies. At the very least, it is at odds with the idea that the realm of contingency forms a single integrated system—a tree-like structure in which the connecting links are dependencies and in which every node is linked to others. There is a potential place in such a structure for initial nodes—nodes ancestrally dependent on no others, which thus correspond to brute contingencies—and for terminating branches, whose terminal nodes thus correspond to barren contingencies. What there is no place for is a node that is both initial and terminal; for by forgoing any relations of dependency, such a node forfeits any case to be regarded as part of the structure at all.

What should we conclude? The picture of the realm of contingency as comprising such a unified web seems to go deep enough in our ordinary thinking to ensure that it will be very difficult to vindicate the conception of mathematical objects as strongly contingent merely by description of anything we already find intuitive. But it is hard to avoid feeling that there is a deeper incongruity in any play with absolutely insular contingency, yet to be brought out. At the very least, a supporter of Field who goes in this direction has to swallow some very lumpy metaphysics for the sake of his ontological conscience.

The main purpose of this paper has been to raise the question whether someone who takes a nominalist-cum-literalist position has the resources coherently to classify the modal status of the proposition that there are numbers, or abstract mathematical objects generally. By contrast, the form of deflationary Platonism—Fregean Platonism, or "Platonism for cheap," as Field styles it—whose claims we have each defended elsewhere, does, we believe, provide a coherent and coherently argued view: it is that the existence of numbers is a necessity, whose truth may be recognized a priori. But the further

defense of that contention is for another occasion: in particular, we ought to stress that the argument of this paper is not, and is not meant to be, a positive argument for this, or any other, version of Platonism.[27] On the contrary, it relies upon no assumption that could not be endorsed by an adherent of classical reductive nominalism, formalism, or—so far as we can see—virtually any other position that stands opposed to the unorthodox variety of nominalism against which it is directed.

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[27] Experience has taught us that there is a tendency toward misunderstanding of this point. So it may help to stress what sort of view comes within the range of our argument. Not any form of anti-Platonism but only views which specifically allow four things:

(i) That the sentences of number theory, analysis, etc., have sufficiently determinate truth conditions to give sense to the question whether they are satisfied or not;
(ii) that those truth conditions are correctly reflected by the overt syntax of the sentences in question;
(iii) that those truth conditions are systematically unrealized; and
(iv) that they are so as a matter either of conceptual necessity or strong contingency.

—only an anti-Platonism that grants *all* these is in trouble if the argument of the paper fully succeeds. Our discussion, if cogent, ought therefore to leave most anti-Platonists undismayed, just because the effect of the most widely influential anti-Platonist arguments—in particular, those deriving from causal accounts of singular reference and Benacerrafian worries about the inscrutability of reference of mathematical singular terms—is (or ought to be) to call one or more of these presuppositions into question. Benacerraf himself, for instance, rejected clause (ii); and section III argued in effect that one who holds that reference to abstract objects is necessarily precluded by their causal inertness cannot coherently endorse both (i) and (ii).

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[Footnotes]

[6] **Perception and Mathematical Intuition**
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