HAIRIER THAN PUTNAM THOUGHT

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FROM one point of view, it may seem that Sorites Paradoxes are a consequence of the very nature of vagueness. If 'bald', for example, lacks sharp boundaries, that can only mean, it appears, that there is no determinate number of hairs, n, such that any man with n hairs on his head is bald while some man with $n + 1$ hairs on his head is not. Accordingly it seems we must deny

(1) $(\exists n)(\mathrm{BM}(n) \,\&\, {\sim}\mathrm{BM}(n+1))$

(where 'BM(n)' is short for 'Any man with n hairs on his head is bald'). Classically this denial is equivalent to assertion of

(2) $(n)(\mathrm{BM}(n) \rightarrow \mathrm{BM}(n+1))$,

which generates a Sorites paradox. Conjoined, for example, with the undeniable

(3) BM(0),

(2) will lead, either by successive applications of universal instantiation and detachment, or by induction, to

(4) BM(50,000)

which is clearly false. If we essay to treat this result as a *reductio* of (2), we wind up with

(5) ${\sim}(n)(\mathrm{BM}(n) \rightarrow \mathrm{BM}(n+1))$

which is classically equivalent to (1). Hence, it appears, if any predicate is vague, it must be Sorites-generating (or *tolerant*); and the option of treating Sorites paradoxes as a *reductio* of their major premises is available only at the cost of denying the vagueness of the predicates involved – which is hardly less paradoxical.

In 'Vagueness and Alternative Logic' (*Realism and Reason*, Cambridge 1983, pp. 271-86, especially 285-6), Hilary Putnam puts forward a suggestion for a formal treatment of the logic of vagueness. His leading thought, although not presented in the context of exactly the foregoing perspective on the problem, is in effect that the equivalence of (5) with (1) is not intuitionistically valid. May not a suitable treatment for vague predicates be available by treating them in the manner in which intuitionistic logic treats predicates in number theory which are not effectively decidable? The prospect is thereby opened of treating the paradox as indeed a *reductio* of (2), so accepting (5), without commitment to (1) with its paradoxical imputation of precision to 'BM(x)'.

Putnam admits that, at the time of writing, he had not thought this idea through. What will already be apparent to the alert reader is that, in order to disclose serious difficulties for the proposal, Putnam would not have had to think very far. For if the denial of (1) is indeed a satisfactory expression of the vagueness of 'BM(x)', intuitionistic logic provides all the materials needed for the paradox. This may be seen in either of two ways. The denial of (1) is intuitionistically equivalent both to

(6) $(n)(\mathrm{BM}(n) \rightarrow \sim\sim\mathrm{BM}(n+1))$

and to

(7) $(n)[\sim\mathrm{BM}(n+1) \rightarrow \sim\mathrm{BM}(n)]$

Now appropriately many steps of contraposition (which is of course intuitionistically valid) will obtain from (6) any principle of the form

(8) $(n)[\sim^{k}\mathrm{BM}(n) \rightarrow \sim^{k+2}\mathrm{BM}(n+1)]$

(where '$\sim^{k}\mathrm{BM}(n)$' is to indicate the result of prefixing 'BM(n)' by some even number, k, of occurrences of '~'). By appropriate successive appeal, then, to the members of the series of such principles, there will be no difficulty (in principle) in advancing from (3) and (6) as initial premises to, say

(9) $\sim^{100{,}000}\mathrm{BM}(50{,}000)$.

But the admitted falsity of (4) yields

(10) $\sim\mathrm{BM}(50{,}000)$

which in turn entails

(11) $\sim(\sim^{99{,}998}\mathrm{BM}(50{,}000))$

of which (9) is the negation. Alternatively, and more simply, (7) will generate a Sorites paradox back-to-front, as it were, starting from (10) and leading to the denial of (3). So the paradox is with us yet.

Although Putnam's proposal is unsatisfactory, its failure is instructive: if the *denial* of principles like (1) is considered to follow from the vagueness of the relevant expressions, then no logic which, like intuitionistic logic, endorses *reductio ad absurdum* and the classical handling of the quantifiers and conjunction can prevent the derivation of Sorites paradoxes. This is evident from the starkest form of the paradox, a back-to-front version each phase of which proceeds as follows:

i	(i)	$\sim(\exists n)[\mathrm{BM}(n) \,\&\, \sim\mathrm{BM}(n+1)]$	Assumption
ii	(ii)	$\sim\mathrm{BM}(k+1)$	Assumption
iii	(iii)	$\mathrm{BM}(k)$	Assumption
ii, iii	(iv)	$\mathrm{BM}(k) \,\&\, \sim\mathrm{BM}(k+1)$	(ii), (iii), &I
ii, iii	(v)	$(\exists n)[\mathrm{BM}(n) \,\&\, \sim\mathrm{BM}(n+1)]$	(iv), EI
i, ii	(vi)	$\sim\mathrm{BM}(k)$	(iii), (i)/(v), RAA

Certain Relevant Logics do indeed, of course, modify the appropriate classical rules.[1] But intuitionistic logic doesn't. Moreover, it is difficult to see how an intuitionistic semantics for negation, appropriately generalized to handle non-mathematical contexts, could fail to endorse the negation of (1). For, intuitionistically, one is entitled to assert the negation of a statement just in case one has verified that no constructive proof of that statement is in principle possible; and to recognize the vagueness of 'BM(x)' is surely exactly to recognize that all possibility of a *constructive* verification of (1) can be discounted.

Suppose it agreed that it cannot be a satisfactory response to the paradox to admit the inconsistency of vague expressions. What we have seen is that the *denial* of (1) cannot be, for anyone with classical or intuitionistic sympathies, a satisfactory way of expressing the vagueness of 'BM(x)'; and, as noted, the *assertion* of (1) merely flies in the face of its vagueness. It follows that a satisfactory account of the matter must render it coherent to hold that (1) may correctly be neither asserted nor denied; although we may take the view that its negation, and (2), may, and indeed must, be denied, since refuted by the paradox. Consequently the unrestricted validity of Double Negation Elimination for statements containing vague expressions is indeed in doubt, and Putnam's proposal has point to that extent. But only to that extent. For, as noted, the motivation for intuitionistic-type restrictions in the logic of vague expressions cannot closely resemble orthodox intuitionistic semantics if we are to avoid denial of (1); and, further, until a way has been found of properly acknowledging vagueness while avoiding such a denial, recourse to intuitionistic logic is no help.[2]

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[1] The central tenet of relevant logic is that the mere lack of a counterexample does not guarantee an entailment. Hence, (i) will not, without qualification, entitle us to pass from (ii) to (vi). Note that this move would be an instance of *modus ponendo tollens*, classically and relevantly equivalent to *modus tollendo ponens*, i.e. Disjunctive Syllogism, which is, familiarly, relevantly invalid. What *in detail* a 'relevantist' will demur at in the reasoning is another matter.

[2] We are indebted to Peter Clark and John Slaney for their contributions to the discussion which suggested this note.